

CSE 143

Lecture 23

Hashing

read 11.2

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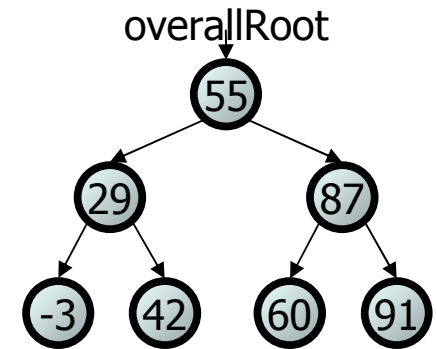
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SearchTree as a set

- We implemented a class `SearchTree` to store a BST of `ints`:
- Our BST is essentially a set of integers.

Operations we support:

- `add`
- `contains`
- `remove`
- ...



- But there are other ways to implement a set...

How to implement a set?

- Elements of a `TreeSet` (`IntTree`) are in BST sorted order.
 - We need this in order to add or search in $O(\log N)$ time.
- But it doesn't really matter what order the elements appear in a set, so long as they can be added and searched quickly.
- Consider the task of storing a set in an array.
 - What would make a good ordering for the elements?

index	0	1	2	3	4	5	6	7	8	9
value	7	11	24	49	0	0	0	0	0	0

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	0	0	7	0	49

Hashing

- **hash:** To map a value to an integer index.
 - **hash table:** An array that stores elements via hashing.
- **hash function:** An algorithm that maps values to indexes.
 - one possible hash function for integers: **$\text{HF}(\mathbf{I}) \rightarrow \mathbf{I} \% \text{length}$**

```
set.add(11);           // 11 % 10 == 1
set.add(49);           // 49 % 10 == 9
set.add(24);           // 24 % 10 == 4
set.add(7);            // 7 % 10 == 7
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	0	0	7	0	49

Efficiency of hashing

```
public static int hashFunction(int i) {  
    return Math.abs(i) % elementData.length;  
}
```

- Add: set `elementData[HF(i)] = i;`
- Search: check if `elementData[HF(i)] == i`
- Remove: set `elementData[HF(i)] = 0;`

- What is the runtime of `add`, `contains`, and `remove`?
– **$O(1)$!**

- Are there any problems with this approach?

Collisions

- **collision:** When hash function maps 2 values to same index.

```
set.add(11);  
set.add(49);  
set.add(24);  
set.add(7);  
set.add(54); // collides with 24!
```

- **collision resolution:** An algorithm for fixing collisions.

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	54	0	0	7	0	49

Probing

- **probing**: Resolving a collision by moving to another index.
 - **linear probing**: Moves to the next index.

```
set.add(11);  
set.add(49);  
set.add(24);  
set.add(7);  
set.add(54); // collides with 24; must probe
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	54	0	7	0	49

- Is this a good approach?
 - variation: **quadratic probing** moves increasingly far away

Clustering

- **clustering**: Clumps of elements at neighboring indexes.
 - slows down the hash table lookup; you must loop through them.

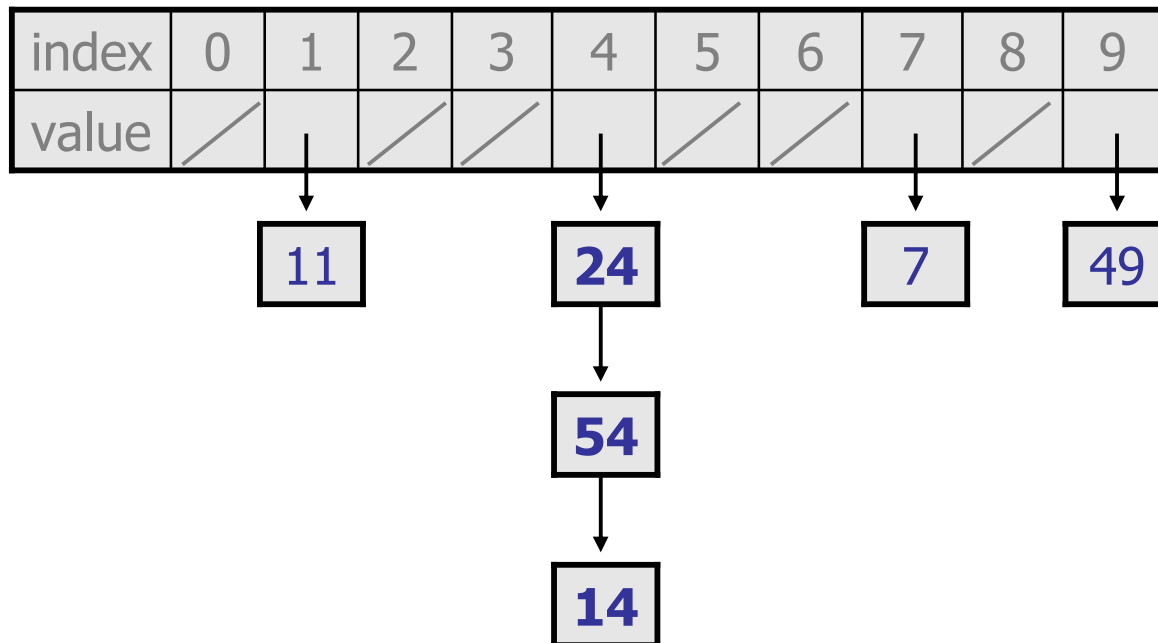
```
set.add(11);  
set.add(49);  
set.add(24);  
set.add(7);  
set.add(54); // collides with 24  
set.add(14); // collides with 24, then 54  
set.add(86); // collides with 14, then 7
```

index	0	1	2	3	4	5	6	7	8	9
value	0	11	0	0	24	54	14	7	86	49

- Now a lookup for 94 must look at 7 out of 10 total indexes.

Chaining

- **chaining:** Resolving collisions by storing a list at each index.
 - add/search/remove must traverse lists, but the lists are short
 - impossible to "run out" of indexes, unlike with probing



Hash set code

```
import java.util.*;    // for List, LinkedList
public class HashIntSet {
    private static final int CAPACITY = 137;
    private List<Integer>[] elements;

    // constructs new empty set
    public HashSet() {
        elements = (List<Integer>[]) (new
List[CAPACITY]);
    }

    // adds the given value to this hash set
    public void add(int value) {
        int index = hashFunction(value);
        if (elements[index] == null) {
            elements[index] = new
LinkedList<Integer>();
        }
        elements[index].add(value);
    }
}
```

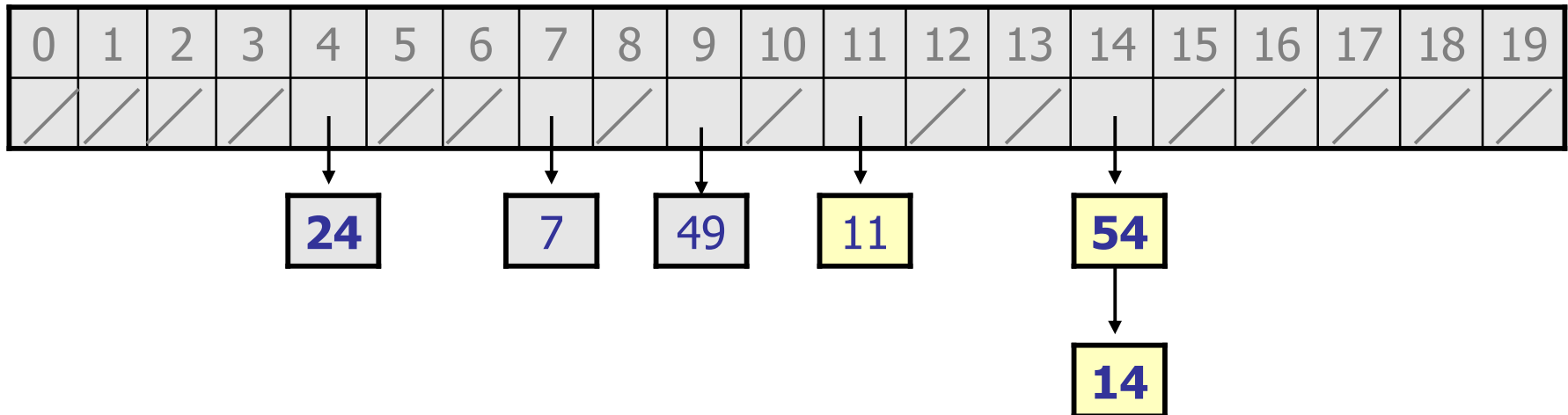
Hash set code 2

```
...
// Returns true if this set contains the
given value.
public boolean contains(int value) {
    int index = hashFunction(value);
    return elements[index] != null &&
        elements[index].contains(value);
}

// Removes the given value from the set, if
it exists.
public void remove(int value) {
    int index = hashFunction(value);
    if (elements[index] != null) {
        elements[index].remove(value);
    }
}
}
```

Rehashing

- **rehash:** Growing to a larger array when the table is too full.
 - Cannot simply copy the old array to a new one. (Why not?)
- **load factor:** ratio of (*# of elements*) / (*hash table length*)
 - many collections rehash when load factor $\cong .75$
 - can use big prime numbers as hash table sizes to reduce collisions

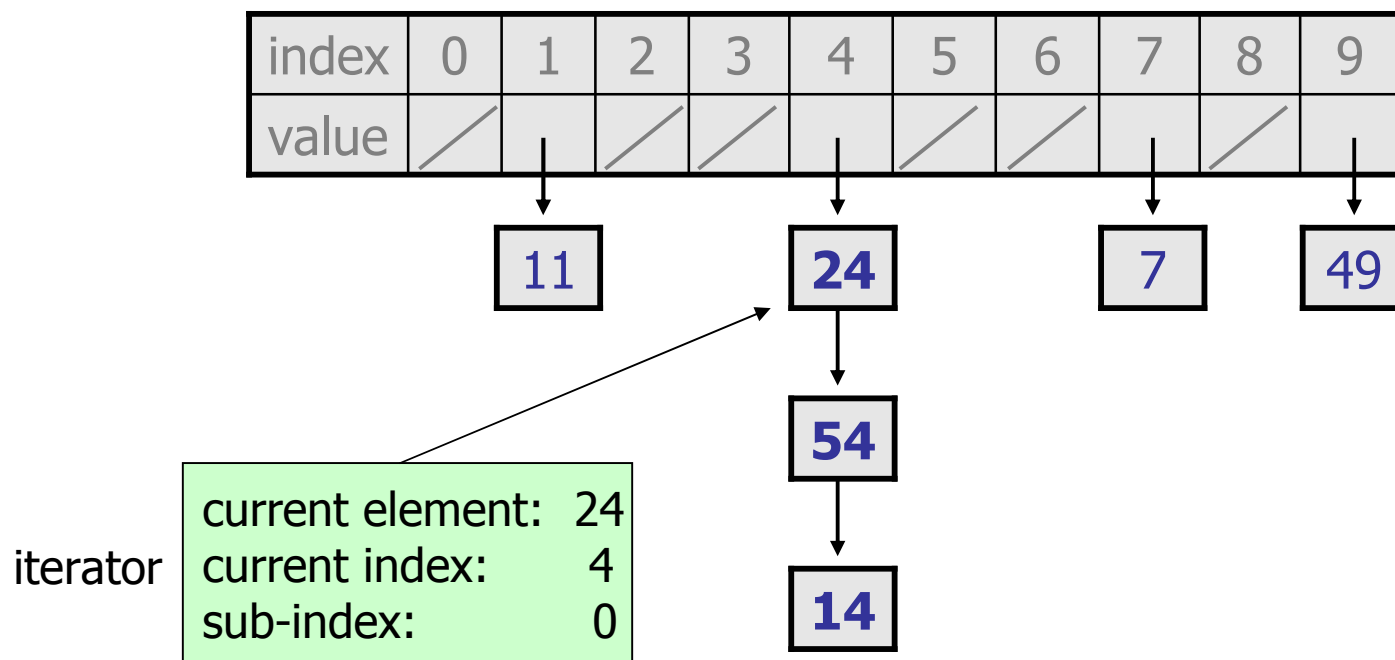


Rehashing code

```
...  
    // Grows hash array to twice its original  
size.  
    private void rehash() {  
        List<Integer>[] oldElements = elements;  
        elements = (List<Integer>[])  
            new List[2 *  
elements.length];  
        for (List<Integer> list : oldElements) {  
            if (list != null) {  
                for (int element : list) {  
                    add(element);  
                }  
            }  
        }  
    }  
}
```

Other questions

- How would we implement `toString` on a `HashSet`?
- How would we implement an `Iterator` over a `HashSet`?



Hashing objects

- It is easy to hash an integer I (use index $I \% length$).
 - How can we hash other types of values (such as objects)?
- All Java objects contain the following method:

```
public int hashCode()
```

Returns an integer hash code for this object.
 - We can call `hashCode` on any object to find its preferred index.
- How is `hashCode` implemented?
 - Depends on the type of object and its state.
 - Example: a `String`'s `hashCode` adds the ASCII values of its letters.
 - You can write your own `hashCode` methods in classes you write.
 - All classes come with a default version based on memory address.

Hash function for objects

```
public static int hashFunction(E e) {  
    return Math.abs(e.hashCode()) % elements.length;  
}
```

- Add: set `elements[HF(o)] = o;`
- Search: check if `elements[HF(o)].equals(o)`
- Remove: set `elements[HF(o)] = null;`

String's hashCode

- The `hashCode` function inside `String` objects looks like this:

```
public int hashCode() {  
    int hash = 0;  
    for (int i = 0; i < this.length(); i++) {  
        hash = 31 * hash + this.charAt(i);  
    }  
    return hash;  
}
```

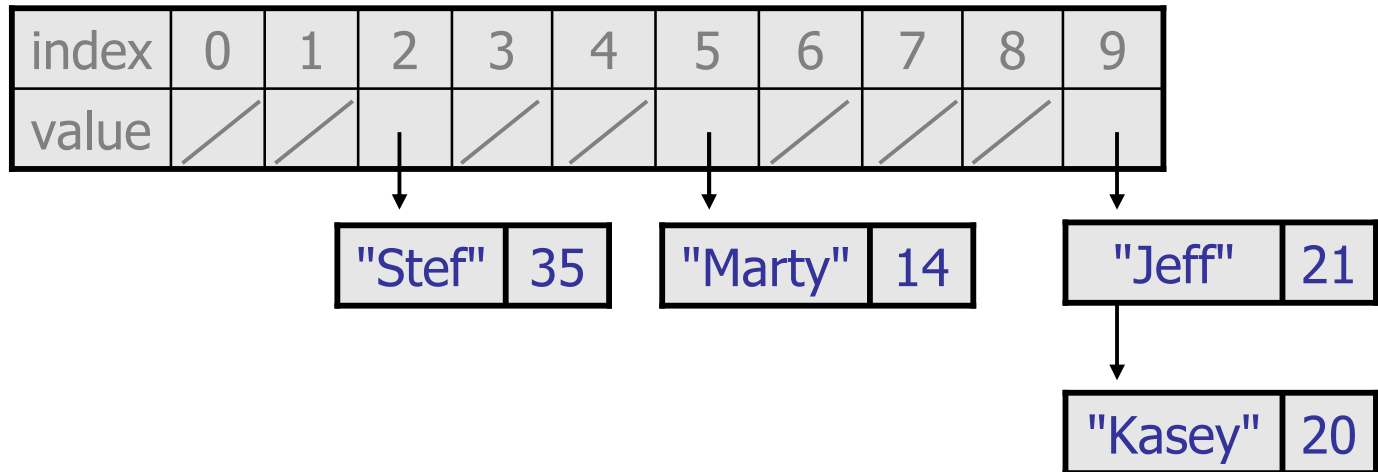
$$h(s) = \sum_{i=0}^{n-1} s[i] \cdot 31^{n-1-i}$$

- As with any general hashing function, collisions are possible.
 - Example: "Ea" and "FB" have the same hash value.
- Early versions of the Java examined only the first 16 characters. For some common data this led to poor hash table performance.

Implementing a hash map

- A hash map is just a set where the lists store key/value pairs:

```
//      key      value
map.put("Marty", 14);
map.put("Jeff", 21);
map.put("Kasey", 20);
map.put("Stef", 35);
```



- Instead of a `List<Integer>`, write an inner `Entry` node class with `key` and `value` fields; the map stores a `List<Entry>`

Implementing a tree map

- Similar to difference between `HashMap` and `HashSet`:
 - Each node now will store both a key and a value
 - tree is BST ordered by keys
 - keys must be `Comparable`

