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Student ID: ______________________________________

<u>SOLUTIONS</u>

<u>CSE 311 Winter 2011: Sample Midterm Exam</u>

(closed book, closed notes except for 1-page summary)
Total: 100 points, 5 questions. Time: 50 minutes

<u>Instructions</u>:

1. Write your name and student ID on the first sheet (once you start, write your last name on all sheets). Write or mark your answers in the space provided. If you need more space or scratch paper, additional sheets will be available. Make sure you write down the question number and your name on any additional sheets.
2. <u>Tables for logical equivalence and set identities</u> are included in the back.
3. Read all questions carefully before answering them. Feel free to come to the front to ask for clarifications.
4. *Hint 1*: You may answer the questions in any order, so if you find that you're having trouble with one of them, move on to another one that seems easier.
5. *Hint 2*: If you don't know the answer to a question, don't omit it - do the best you can! You may still get partial credit for whatever you wrote down. Good luck!

<u>Do not start</u> until you are given the "green signal"…

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1. (20 points: 4 points each) Circle True (T) or False (F) below. Very briefly justify your answers (e.g. by giving an example or counter-example, by citing a theorem or result we proved in class, or by *briefly* sketching a proof or construction).

a. For all propositional variables p and q, $(p \vee q) \rightarrow p$ is a tautology..................T F
Why/Why not?
FALSE. $(p \vee q) \rightarrow p$ is not a tautology because it is not always true. For p = F and q = T, $(p \vee q) \rightarrow p$ is False.

b. $\exists n \in Z\ \forall m \in Z\ (nm = m)$ ....T F
Why/Why not?
TRUE. Choose n = 1. Then nm = m for all integers m.

c. In a vanpool of eight people, at least two were born on the same day of the week..T F
Why/Why not?
TRUE. Proof by contradiction. Assume all were born on different days of the week. This implies 8 distinct days in a week, contradicting the fact that there are 7 days in a week. Therefore, at least 2 were born on same day of week.

d. The function f(x,y) = x-y from $Z \times Z$ to Z is a bijection...................................T F
Why/Why not?
FALSE. f(1,0) = 1 and f(2,1) = 1. f is not 1-1, therefore not a bijection.

e. For the hashing function h(k) = k mod 8, the keys 10 and 20 result in a collision……………………………………………………………………T F
Why/Why not?
FALSE. h(10) = 10 mod 8 = 2 and h(20) = 20 mod 8 = 4. Therefore, there is no collision.

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2. (20 points: 10 points each) Propositional Logic

a. Show that $\neg(p \leftrightarrow q) \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$ using known logical equivalences (see tables at back of this exam).

Example proof: (other proofs exist as well)

$\neg(p \leftrightarrow q)$	$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$	**by Table 8**
	$\equiv \neg(p \rightarrow q) \vee \neg(q \rightarrow p)$	**by De Morgan's law**
	$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$	**by Table 7**
	$\equiv (\neg\neg p \wedge \neg q) \vee (\neg\neg q \wedge \neg p)$	**by De Morgan's law**
	$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$	**by Double negation law**

b. Use a truth table to show that $\neg p \rightarrow (p \rightarrow q)$ is a tautology.

See solution for Exercise 9c in Section 1.2 in the text.

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3. (18 points: 10 and 8 points) Predicate Logic

a. Let S(x) be the predicate “x is a student,” F(x) the predicate “x is a faculty member,” and A(x,y) the predicate “x has asked y a question,” where the domain is the set of all people in CSE. Use only the quantifiers ∃ and ∀ to express the following statements:

   i. There is a faculty member who has never been asked a question by any students.
   $\mathbf{\exists x(F(x) \land \forall y(S(y) \rightarrow \neg A(y,x))}$

   ii. All faculty members have been asked a question by at least two students.
   $\mathbf{\forall x(F(x) \rightarrow \exists y \exists z(y \neq z \land S(y) \land S(z) \land A(y,x) \land A(z,x))}$

b. Express the negations of each of these statements so that all negation symbols immediately precede predicates:

   i. $\forall x \exists y(P(x,y) \land \exists z R(x,y,z))$
   ii. $\forall x \exists y(P(x,y) \rightarrow Q(x,y))$

See solutions for Exercises 31 c and d in Section 1.4 in the text.

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4. (22 points: 12 and 10 points) Rules of Inference and Proofs

a. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.
See solution for Exercise 27 in Section 1.5 in the text.

b. Use a proof by contraposition to show that for all real numbers x, y, if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$.
See solution for Exercise 15 in Section 1.6 in the text.

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5. (20 points: 12 and 8 points) Sets and Number Theory

a. Let A and B be two sets such that $A \subseteq B$. Let U be the universal set.

i. Draw two Venn diagrams, one showing A and B, and another showing $\overline{A}$ and $\overline{B}$.

See textbook/lecture notes for examples.

ii. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

See solution for Exercise 31 in Section 2.2 in the text.

b. State whether the following statements are true or false (no proofs needed):

i. If $n \mid x$, then x **mod** $n = 0$ for any integer x and any positive integer n.
TRUE

ii. -7 **div** 3 = -2
FALSE

iii. $11 \equiv 19 \pmod 4$
TRUE

iv. The number 8 is among the numbers generated by the linear congruential pseudorandom number generator: $x_{n+1} = 3x_n$ **mod** 11 with seed $x_0 = 2$.
TRUE

END OF EXAM

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TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

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TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditionals.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

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TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution