CSE 311 Quiz Section: April 11, 2013 (Solutions)

1. Sum-Of-Products

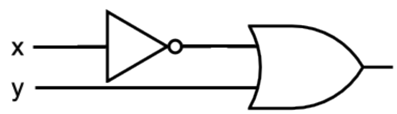
Find the sum-of-products expansion of the Boolean function $F(w,x,y,z)$ that has the value 1 if and only if an odd number of w, x, y, z have value 1.

Solution:
$F(w, x, y, z) = w'xyz + wx'yz + wxy'z + wxyz' + w'x'y'z + w'x'yz' + w'xy'z' + wx'y'z'$

2. Circuits

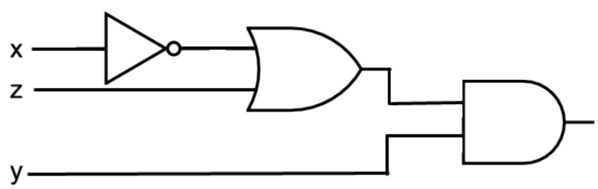
Construct circuits from inverters, AND gates, and OR gates to produce these outputs. Can you simplify any of them? (Note: A bar above an expression means its negation.)

(a) $\bar{x} + y$
Solution:



(b) $xyz + \bar{x}y$
Solution:
We can first simplify by performing the following:

$$\begin{aligned} xyz + \bar{x}y &= y(xz + \bar{x}) \\ &= y[(x + \bar{x})(z + \bar{x})] && \text{(by applying distr. law)} \\ &= y(z + \bar{x}) && \text{(by negation law)} \end{aligned}$$

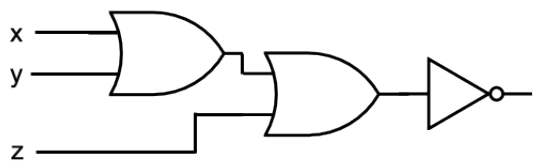


(c) $(\overline{x+y})(\overline{y+z})(\overline{x+z})$

Solution:

We can first simplify by performing the following:

$$\begin{aligned}(\overline{x+y})(\overline{y+z})(\overline{x+z} &= (\bar{x}\bar{y})(\bar{y}\bar{z})(\bar{x}\bar{z}) && \text{(by DeMorgan's Law)}\\ &= \bar{x}\bar{y}\bar{z} && \text{(by idempotent laws)}\\ &= \overline{(x+y+z)} && \text{(by DeMorgan's law)}\end{aligned}$$



3. Translation with Quantifiers

Let $L(x, y)$ be the statement "x loves y." Let the domain for both x and y consist of all people in the world.

(a) There is somebody whom everybody loves.

Solution: You can think of this statement as "There exists a person that each person in the world loves." This makes it easier to translate to the correct logical statement: $\exists y \forall x (L(x, y))$

Note: Order matters with quantifiers. $\forall x \exists y (L(x, y))$ means "Each person in the world loves at least one person in the world" (possibly just himself or herself) or, more fluently, "Everybody loves someone."

(b) Nobody loves everybody.

Solution: You can rephrase this English statement to be "Every person in the world does not love at least one person." Thus our correct logical statement is $\forall x \exists y (\neg L(x, y))$.

Note: Another acceptable solution is $\neg \exists x \forall y (L(x, y))$ or "There does not exist a person that loves all people in the world" which is equivalent to $\forall x \exists y (\neg L(x, y))$ by DeMorgan's Law for quantifiers.

(c) Everyone loves himself or herself.

Solution: $\forall x L(x, x)$

(d) There is someone who loves no one besides himself or herself.

Solution: There are two ways to interpret this statement (we would accept either one): either that the statement implies that the person loves himself or herself, or we simply do not know whether they love themselves or not.

In the first case, we assume $L(x, x)$ is true. Therefore the correct logical statement would be: $\exists x \forall y (L(x, y) \leftrightarrow x = y$ or, equivalently, $\exists x \forall y (\neg L(x, y) \leftrightarrow x \neq y)$.

In the second case, we do not presume that a person loves themselves, and therefore the statement would be: $\exists x \forall y (x \neq y \rightarrow \neg L(x, y))$.

One final note on this one. In English the meaning of these words and the relationship between 'love', 'like', and 'don't care' are more complicated, but in logic these are unrelated predicates unless we add more information. So for this problem it is safe to assume that "x does not love y" $= \neg$ "x loves y" and "x loves y" $= \neg$ "x does not love y".

NOTE: If you want some more guidance on these, you can find some useful tips here:
`http://cstl-cla.semo.edu/hill/pl120/notes/relational%20predicates.htm`