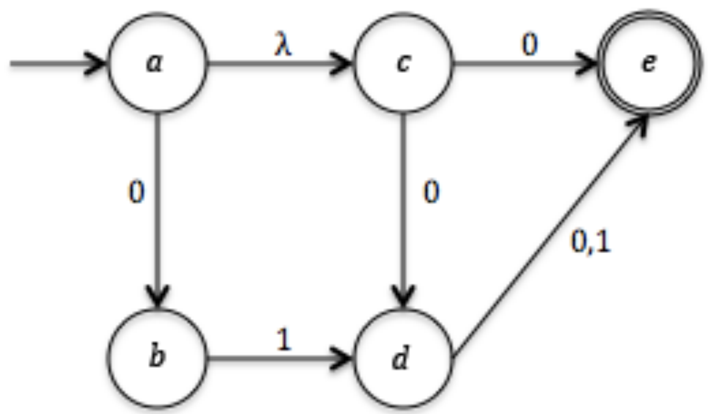
Section Week 9 Worksheet

Solutions

1) NFA to DFA

Convert the following NFA to DFA.



New start state: {a,c}

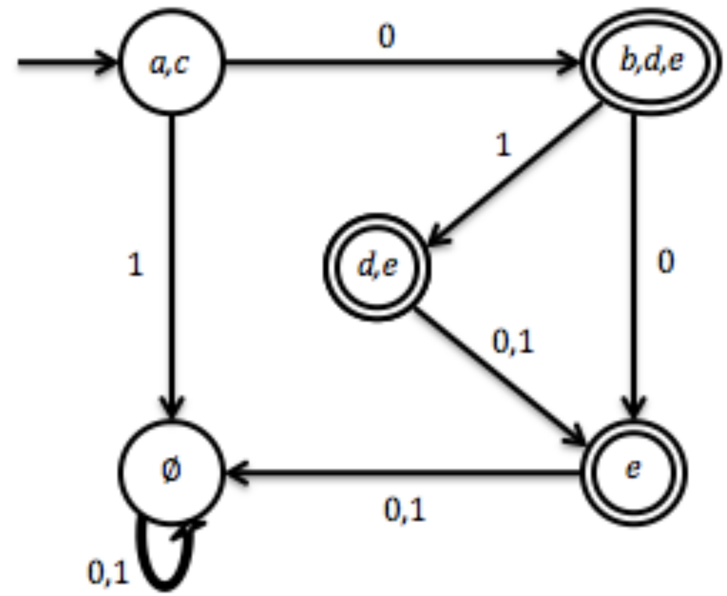
{a,c}→{b,d,e} with a 0
{a,c}→null with a 1

1) NFA to DFA

{b,d,e}→{e} with a 0
{b,d,e}→{d,e} with a 1

{d,e}→{e} with a 0
{d,e}→{e} with a 1

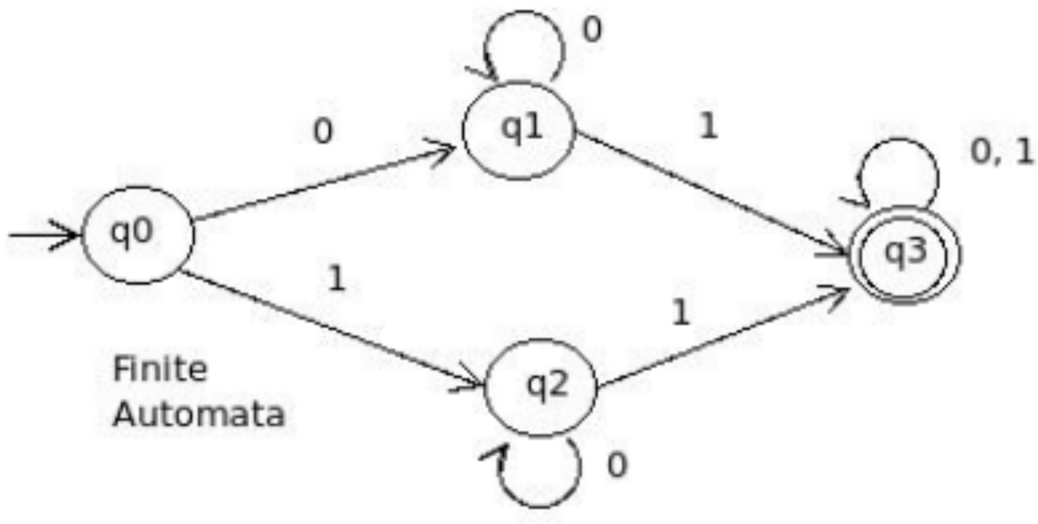
{e}→null with a 0
{e}→null with a 1



Final states (all that contain an e): {b,d,e}, {d,e}, {e}

2) State Minimization

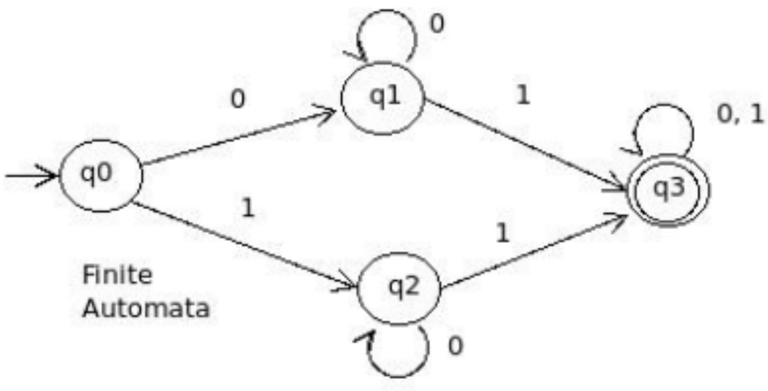
Minimize the following DFA:



Start by dividing into final and non-final states:

G1: $\{q_0, q_1, q_2\}$ G2: $\{q_3\}$

2) State Minimization



G1: {q_0,q_1,q_2} G2: {q_3}

q_0 →G1 with 0

q_0 →G1 with 1

q_1 →G1 with 0

q_1 →G2 with 1

q_2 →G1 with 0

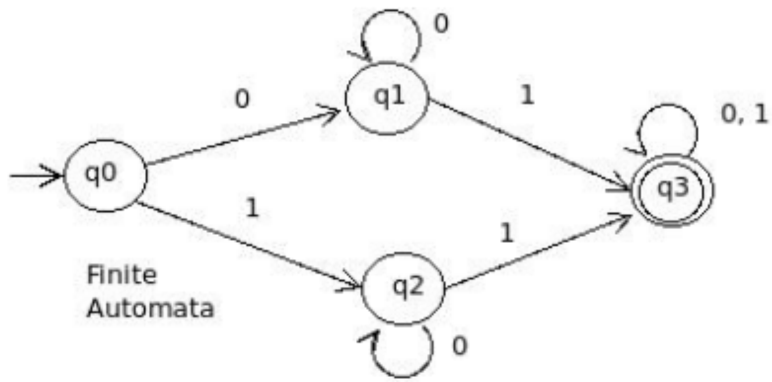
q_2 →G2 with 1

q_3 →G2 with 0

q_3 →G2 with 1

New groups: {q_0}, {q_1, q_2}, {q_3}

2) State Minimization



G1:{q_0} G2: {q_1, q_2} G3: {q_3}

q_0 →G2 with 0

q_0 →G2 with 1

q_1 →G2 with 0

q_1 →G3 with 1

q_2 →G2 with 0

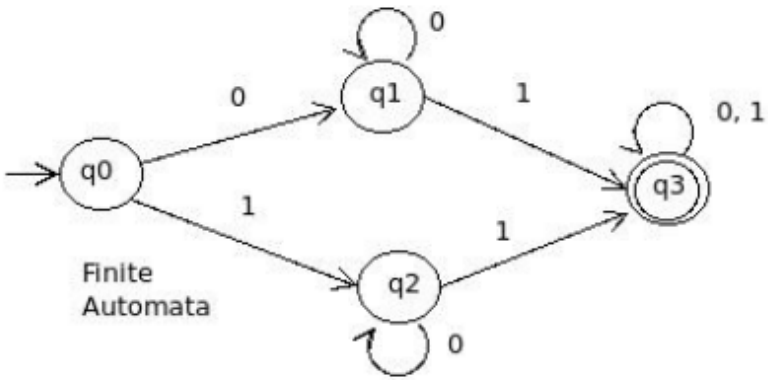
q_2 →G3 with 1

q_3 →G3 with 0

q_3 →G3 with 1

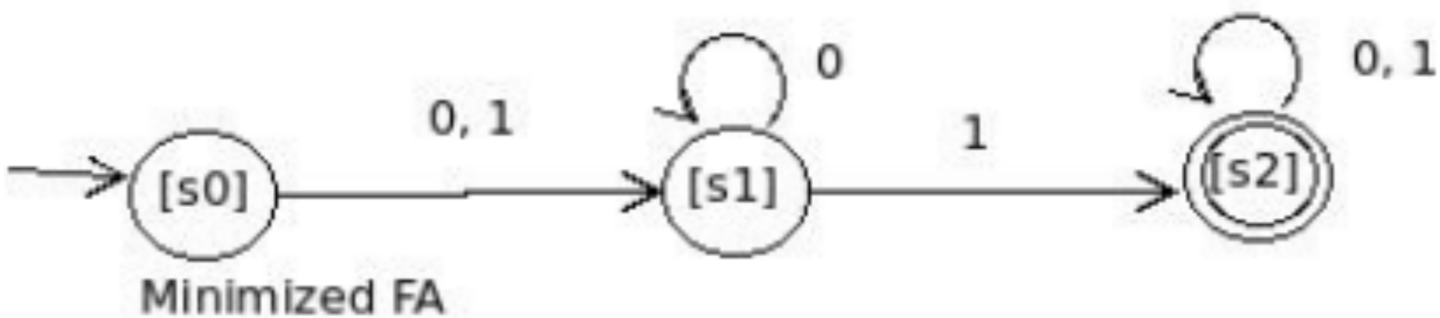
Groups don't change! Thus we are done.

2) State Minimization



$[s_0]$:$\{q_0\}$ $[s_1]$: $\{q_1, q_2\}$ $[s_2]$: $\{q_3\}$

Minimized machine:



3) Non-regular Languages

Prove that $\{0^{2n}1^n : n \geq 0\}$ is non-regular.

Proof: We will show that this language is non-regular by showing that there cannot exist a DFA that accepts it.

First, assume that such a DFA *does* exist that accepts this language, and that it has A states. (A must be a finite number because DFAs are finite– it's even part of their name!)

Next, consider the set of strings $\{\lambda, 00, 0000, \ldots\}$, i.e. $\{0^{2k} : k \geq 0\}$.

This set is infinite, so we must have some strings 0^{2k}, 0^{2j} where $k \neq j$ going to the same state. (Since our DFA has A states and $|S|$ is infinite.)

3) Non-regular Languages

Prove that $\{0^{2n}1^{n} : n \geq 0\}$ is non-regular.

Since $0^{2k}1^{k}$ is in our set, it needs to be accepted by our machine. So we must have a path of 1^{k} that leads to an accept state from our 0^{2k} state.

However, since 0^{2j} and 0^{2k} go to the same state, there is no way for the machine to know which path was taken, so this machine will also have to accept $0^{2j}1^{k}$. Since $j \neq k$, this string is not in our set.

This is a contradiction, thus our assumption is false, meaning there does not exist a FSM that accepts this language. Thus our language is non-regular. ■