Section 4 Worksheet

Solutions

(1) More on Sets

Prove that $A \subseteq B \leftrightarrow B' \subseteq A'$.

Proof: ($\rightarrow$) Let $A \subseteq B$. Assume an element x is a member of B'. (We want to show $x \in A'$.)

Then $x \notin B$ by definition of set complements.

Thus $x \notin A$ because $A \subseteq B$ by assumption, and since $x \notin B \rightarrow x \notin A$. (Contrapositive of the definition of subset.)

$\therefore$ $x \in A'$ by definition of set complement.

So $x \in B' \rightarrow x \in A'$, and thus $B' \subseteq A'$ by def. of subset.

(1) More on Sets

Prove that $A \subseteq B \leftrightarrow B' \subseteq A'$.

($\longleftarrow$) Let $B' \subseteq A'$. Assume an element x is a member of A. (We want to show $x \in B$.)

Then $x \notin A'$ by definition of set complements.

Thus $x \notin B'$ because $B' \subseteq A'$ by assumption, and since $x \notin A' \rightarrow x \notin B'$. (Contrapositive of the definition of subset.)

$\therefore$ $x \in B$ by definition of set complement.

So $x \in A \rightarrow x \in B$, and thus $A \subseteq B$ by def. of subset.

(1) More on Sets

Prove that $A \subseteq B \leftrightarrow B' \subseteq A'$.

We have shown $A \subseteq B \rightarrow B' \subseteq A'$ and $B' \subseteq A' \rightarrow A \subseteq B$, thus we have proven $A \subseteq B \leftrightarrow B' \subseteq A'$. ■

(2) Functions

A = {x : x $\in$ **R**, x≥1}
B = {x : x $\in$ **R**, 0≤x≤1}
C = {x : x $\in$ **R**, -1≤x≤1

(i) $f : A \to B, f(x) = \dfrac{1}{x}$

One-to-one, but not onto.
(0 $\in$ B, but we can never get 0 by plugging in any value in our domain.)

(2) Functions

A = {x : x $\in$ **R**, x≥1}
B = {x : x $\in$ **R**, 0≤x≤1}
C = {x : x $\in$ **R**, -1≤x≤1

(ii) $f : B \rightarrow C, f(x) = x^2$

One-to-one, but not onto.
(-1$\in$C, but we can never get -1 by plugging in any value in our domain.)

(2) Functions

A = {x : x $\in$ **R**, x≥1}
B = {x : x $\in$ **R**, 0≤x≤1}
C = {x : x $\in$ **R**, -1≤x≤1

(iii) $f : B \rightarrow B, f(x) = x^2$

Both one-to-one and onto.

(No negatives to worry about in this case, so we don’t have the same problem as before for onto. One-to-one because no two values in the domain produce the same output.)

(2) Functions

A = {x : x $\in$ **R**, x≥1}
B = {x : x $\in$ **R**, 0≤x≤1}
C = {x : x $\in$ **R**, -1≤x≤1

(iv) $f : C \to B, f(x) = x^2$

Onto, but not one-to-one.

(-1 and 1 are both in domain, both produce output of 1.)

(3) Modular Arithmetic

Find an integer *a* such that:

(i) $a \equiv 43 \pmod{23}$, $-22 \leq a \leq 0$

$a = -3$ (we can check by seeing that $23 | (43-(-3))$)

Def: Let *a*,*b* be integers, and *m* be a positive integer. Then $a \equiv b \pmod{m} \leftrightarrow m | (a-b)$.

(3) Modular Arithmetic

Find an integer *a* such that:

(i) $a \equiv 17 \pmod{29}$, $-14 \le a \le 14$

$a = -12$

(Check: $29 | -29$ ✓)

(3) Modular Arithmetic

Find an integer a such that:

(i) $a \equiv -11 \pmod{21}$, $90 \leq a \leq 110$

$a = 94$

(Check: $21|105$ ✓)