CSE 311 Quiz Section: May 2, 2013

Proofs by Induction

Say we want to prove that some proposition $P(n)$ is true for each integer $n \geq 1$. A proof by induction consists of five steps:

(1) State what we want to prove: "By induction we will show that $P(n)$ is true for each integer $n \geq 1$.
(2) Base Case: Prove $P(1)$ is true.
(3) Inductive Hypothesis: Assume $P(k)$ is true for some arbitrary integer $k \geq 1$.
(4) Inductive Step: Using our inductive hypothesis, prove $P(k+1)$ is true.
(5) State our Conclusion: "Our result follows from induction."

1.

Find a formula for the following expression, where n is any positive integer:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

Use induction to prove that your formula is correct.

2.

Use induction to prove that for all positive integers n:

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = \frac{(-1)^{n-1} n(n+1)}{2}$$

3.

Prove that for all positive integers n:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2$$

Hint: Try replacing the right hand side of the inequality with something that will make the statement stronger.

Challenge Problem: Horse Paradox

The following "proof" purports to show that all horses are the same color. Where is the error in the proof?

Statement: We will show that for any group of n horses, where n is a positive integer, all of them are the same color.
Base Case: $n = 1$. When there is only one horse in the "group", then clearly all horses in that group have the same color.
Inductive Hypotheis: Assume that for some arbitrary integer k, for any group of k horses, all of them are the same color.
Inductive step: We will prove that an arbitrary group of $k+1$ horses are all the same color. First, remove one horse. By our inductive hypothesis, the remaining group of k horses are all the same color. Next, add it back in and remove a different horse. Again, by our inductive hypothesis, the reamining group of k horses are all the same color. Since each horse we removed was the same color as the group when we removed the other one, all $k+1$ horses are the same color.
Conclusion: Any group of n horses are all the same color for any positive integer n. Therefore, all horses are the same color.