Section 01: Propositional Logic Translation

1. Warm-Up

Translate the English sentences below into symbolic logic.

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

2. Truth Tables

Write a truth table for each of the following:

(a) $(r \oplus q) \vee (r \oplus \neg q)$

(b) $(r \vee q) \rightarrow (r \oplus q)$

(c) $p \leftrightarrow \neg p$

3. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

(a) whenever I walk my dog, I make new friends.

(b) I will drink coffee, if Starbucks is open or my coffeemaker works.

(c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

(d) I can go home only if I have finished my homework.

(e) Having an internet connection is necessary to log onto zoom.

4. I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

(a) p: The sun is out
r: We have class outside

$$p \rightarrow r$$

(b) a: the book has been out for a week.
b: I don't have homework.
c: I have finished reading the book.

$$(a \wedge b) \rightarrow c$$

(c) p: I have read the manual
r: I operate the machine

$$r \to p$$

5. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) I will remember to send you the address only if you send me an e-mail message.

(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

(c) Unless I am trying to type something, my cat is either eating or sleeping.

6. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation.

(b) Fill out a truth table for your expression.

7. ??clusive Or

Exclusive or ($\oplus$) and inclusive or ($\vee$) both can be translated as "or" in English. For each of the following ambiguous phrases, decide which type of "or" is likely meant and why.

(a) Experience with C or Java is required.

(b) Lunch includes soup or salad.

(c) Publish or perish.

(d) To enter the country, you need a passport or voter registration card.

8. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \to r$ vs. $r \to p$

(b) $a \to (b \wedge c)$ vs. $(a \to b) \wedge c$

9. They mean the same thing

In the activity from lecture 2, we showed the following.

$$\neg(q \to r) \equiv \neg(\neg q \vee r)$$

Use the elementary equivalences presented at the end of lecture 2 to argue that the following pairs are equivalent.

$$\neg(\neg q \vee r) \equiv \neg(\neg q) \wedge \neg r \quad (1)$$

$$\neg(\neg q) \wedge \neg r \equiv q \wedge \neg r \quad (2)$$

$$q \wedge \neg r \equiv \neg r \wedge q \quad (3)$$

Your friend says this means that $\neg(q \to r) \equiv \neg r \wedge q$. Is that true?

10. Equivalent Translations

Prove that the following English statements are equivalent.
(i) Unless it isn't raining or I don't have an umbrella, I buy a book.
(ii) It isn't raining or I don't have an umbrella or I buy a book.

11. Circuitous

Translate the following circuit into a logical expression.