# Section 04: Review - English Proofs

## 1. Proof Strategies

**Proof by Counterexample**

To prove $\neg\forall x P(x)$, prove $\exists x \neg P(x)$

- An application of De Morgan's Law
- Intuition: If something is not true for every $x$ in the domain, it must be not true for at least a $x$ in the domain.
- Goal: Identify the $x$ in the domain for which P(x) is not true - this is a *counterexample* to $\forall x P(x)$

**Proof by Contrapositive**

We can prove $p \to q$ by proving the equivalent contrapositive, $\neg q \to \neg p$.

| | | |
|---|---|---|
| 1.1 | $\neg q$ | [Assumption] |
| 1.2 | ... | [???] |
| 1.3 | $\neg p$ | [???] |
| 1. | $\neg q \to \neg p$ | [Direct Proof] |
| 2. | $p \to q$ | [Contrapositive, 1] |

**Proof by Contradiction**

If we show $p \to F$, we have shown $\neg p$.

- This style of proof is generally done by "supposing" the *opposite* of what we want to prove, and showing that under this condition, we see a contradiction.

| | | |
|---|---|---|
| 1.1 | $p$ | [Assumption] |
| 1.2 | ... | [???] |
| 1.3 | $F$ | [???] |
| 2. | $p \to F$ | [Direct Proof] |
| 3. | $\neg p \lor F$ | [Law of Implication] |
| 4. | $\neg p$ | [Identity] |