Relations

Relations

A (binary) relation from A to B is a subset of $A \times B$

A (binary) relation on A is a subset of $A \times A$

Wait what?

$\leq$ is a relation on $\mathbb{Z}$.

"$3 \leq 4$" is a way of saying "3 relates to 4" (for the $\leq$ relation)

(3,4) is an element of the set that defines the relation.

Try a few of your own

Decide whether each of these relations are

Reflexive, symmetric, antisymmetric, and transitive.

$\subseteq$ on $\mathcal{P}(\mathcal{U})$

$\geq$ on $\mathbb{Z}$

$>$ on $\mathbb{R}$

$|$ on $\mathbb{Z}^+$

$|$ on $\mathbb{Z}$

$\equiv (mod\ 3)$ on $\mathbb{Z}$

Symmetry: for all $a, b \in S$, $[(a, b) \in R \rightarrow (b, a) \in R]$

Antisymmetry: for all $a, b \in S$, $[(a, b) \in R \land a \neq b \rightarrow (b, a) \notin R]$

Transitivity: for all $a, b, c \in S$, $[(a, b) \in R \land (b, c) \in R \rightarrow (a, c) \in R]$

Reflexivity: for all $a \in S$, $[(a, a) \in R]$

Directed Graphs

$G = (V, E)$

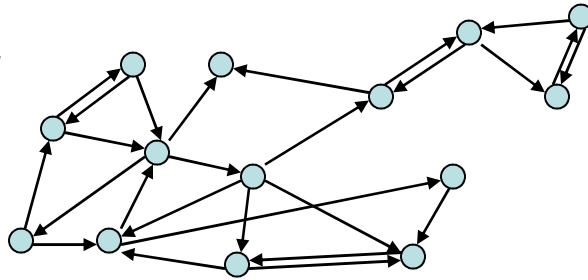V is a set of vertices (an underlying set of elements)

E is a set of edges (ordered pairs of vertices; i.e. connections from one to the next).

Path $v_0, v_1, \ldots, v_k$ such that $(v_i, v_{i+1}) \in E$
Simple Path: path with all v_i distinct
Cycle: path with $v_0 = v_k$ (and $k > 0$)
Simple Cycle: simple path plus edge
(v_k, v_0) with $k > 0$

Relations and Graphs

Describe how each property will show up in the graph of a relation.

Reflexive

Symmetric

Antisymmetric

Transitive