Bijection

One-to-one (aka injection)

A function f is one-to-one iff
$\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

Onto (aka surjection)

A function $f: A \rightarrow B$ is onto iff
$\forall b \in B \exists a \in A(b = f(a))$

Bijection

A function $f: A \rightarrow B$ is a bijection iff
f is one-to-one and onto

A bijection maps every element of the domain to exactly one element of the co-domain, and every element of the domain to exactly one element of the domain.

Some infinite sets

Two sets A, B have the same size (same cardinality) if and only if there is a bijection $f: A \rightarrow B$

Let's compare the sizes of: $\mathbb{N}$, $\mathbb{Z}$, $\{x : x \text{ is an even integer}\}$

Countable

Countable

The set A is countable iff there is an injection from A to $\mathbb{N}$,
Equivalently, A is countable iff it is finite or there is a bijection from A to $\mathbb{N}$

$\mathbb{N}, \mathbb{Z}$, $\{x: x$ is an even integer$\}$ are all countable.

Proof that $[0,1)$ is not countable

Suppose, for the sake of contradiction, that there is a list of them:

Number	Digits after decimal	0	1	2	3	4	5	6	7	…
$f(0)$	0.	3	3	3	3	3				
$f(1)$	0.	2	7	2	7	2				
$f(2)$	0.	1	4	1	5	9				
$f(3)$	0.	2	2	2	2	2				
$f(4)$	0.	1	2	3	4	5	6	7	8	…
$f(5)$	0.	9	8	7	6	5	4	3	2	…
$f(6)$	0.	8	2	7	6	4	5	7	4	…
$f(7)$	0.	5	9	4	2	7	5	1	7	…
				…	…	…	…	…	…	…

Flipping Rule: let's set the i^{th} column to:
7 if $\boldsymbol{f(i)}$'s $\boldsymbol{i^{th}}$ column is not **7**
3 if $\boldsymbol{f(i)'s\ i^{th}}$ column is **7**.

0.73777733…