The Halting Problem

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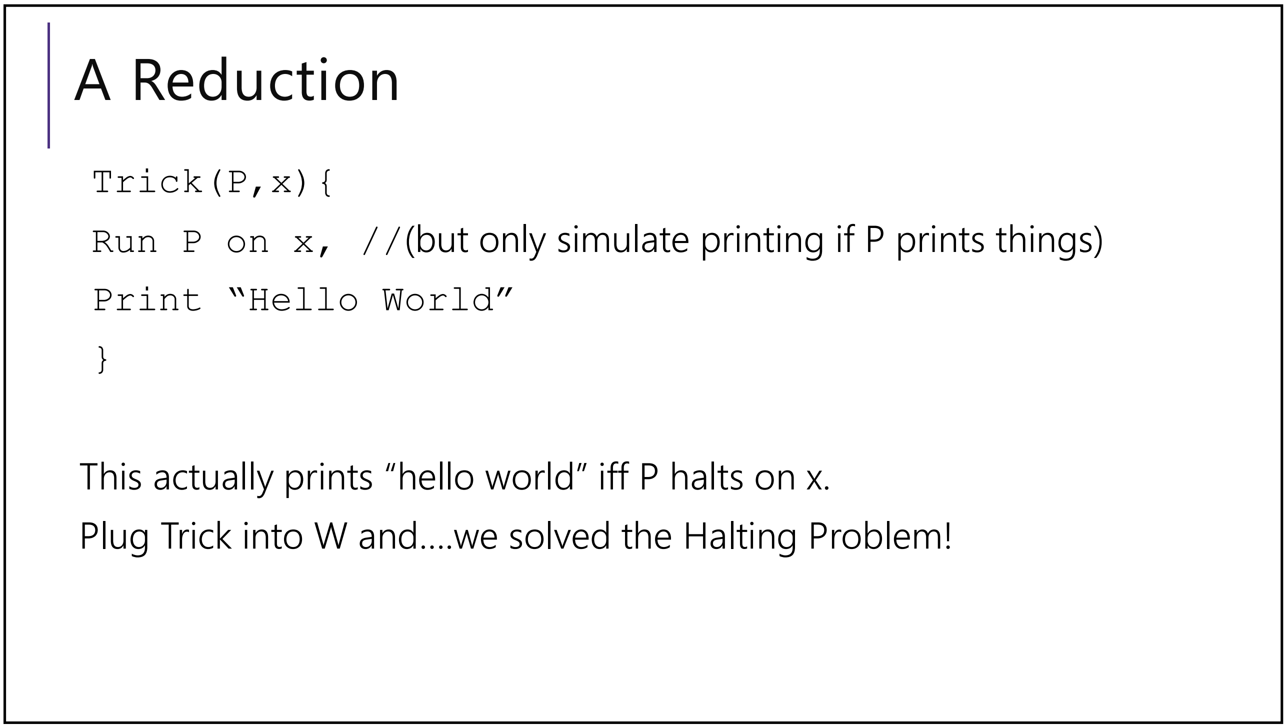
Given: source code for a program P and x an input we could give to P
Return: True if P will halt on x, False if it runs forever (e.g. goes in an infinite loop or infinitely recurses)

This would be super useful to solve!

We can’t solve it…let’s find out why.

A Very Tricky Program.

```
Diagonal.java(String x){
   Run H.exe on input <x, x>
   if(H.exe says “x halts on x”)
       while(true){//Go into an infinite loop
            int x=2+2;
       }
   else //H.exe says “x doesn’t halt on x”
      return; //halt.
}
```

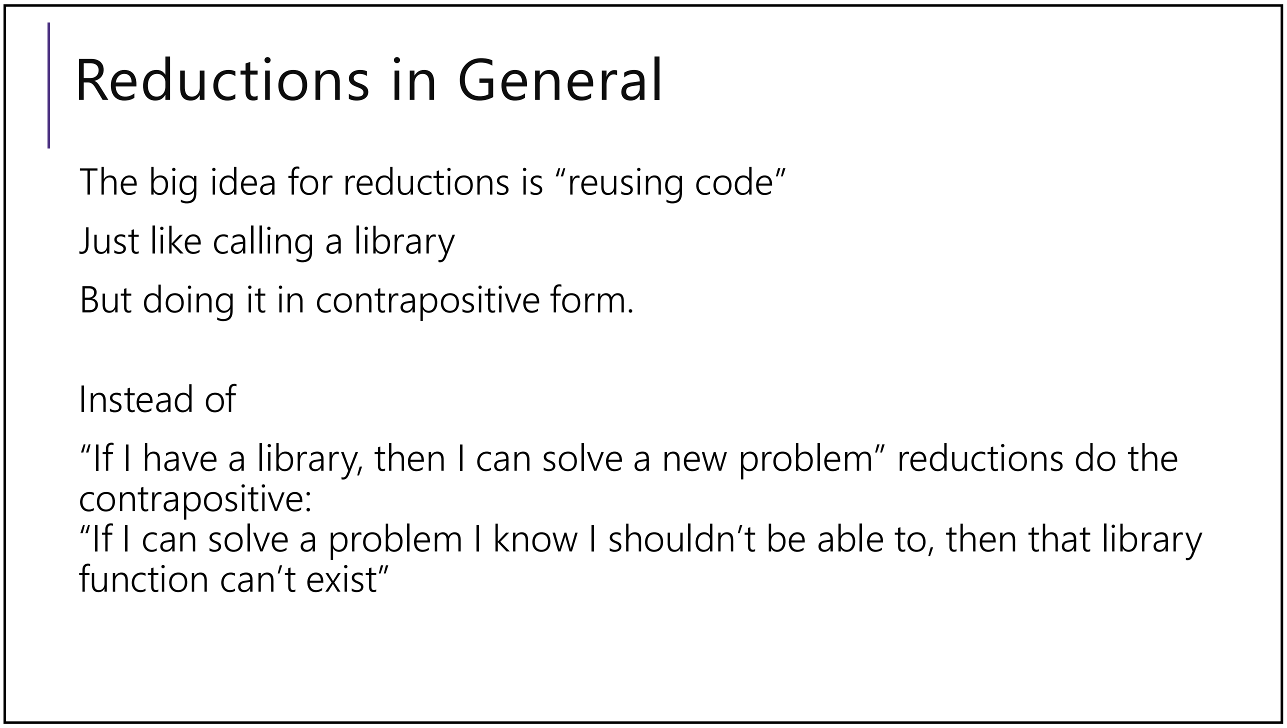


A Reduction

```
Trick(P,x){
Run P on x, //(but only simulate printing if P prints things)
Print "Hello World"
}
```

This actually prints “hello world” iff P halts on x.

Plug Trick into W and….we solved the Halting Problem!



Reductions in General

The big idea for reductions is “reusing code”

Just like calling a library

But doing it in contrapositive form.

Instead of

“If I have a library, then I can solve a new problem” reductions do the contrapositive:
“If I can solve a problem I know I shouldn’t be able to, then that library function can’t exist”