Section 2

CSE 311 - Sp 2022

Administrivia

Announcements and Reminders

- HW1 due yesterday 10PM on Gradescope
  - Remember, you have 6 late days to use throughout the quarter
  - You can use up to 3 late days on any 1 assignment
  - You don’t get extra credit for having any unused late days, so feel free to use them if you need them!

- Check the course website for OH times!
  - There are both zoom and in-person office hours every day

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/22sp/assignments/HowToLaTeX.pdf
- Equivalence Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/reference-logical_equiv.pdf
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/logicalConnectPoster.pdf
- Boolean Algebra Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/22sp/resources/reference-boolean-alg.pdf
- Plus more!

Equivalence & Symbolic Proof

Equivalence Review

$p \wedge (p \rightarrow q)$	$\equiv p \wedge (\neg p \vee q)$	[Law of Implication]
	$\equiv (p \wedge \neg p) \vee (p \wedge q)$	[Distributivity]
	$\equiv F \vee (p \wedge q)$	[Negation]
	$\equiv (p \wedge q) \vee F$	[Commutativity]
	$\equiv p \wedge q$	[Identity]

LaTeX Example

```latex
\begin{align*}
  \neg p \rightarrow (q \rightarrow r) &\equiv \neg \neg p \vee (q \rightarrow r) &&\text{Law Of Impl.} \\
  &\equiv p \vee (q \rightarrow r) &&\text{Double Neg} \\
  &\equiv p \vee (\neg q \vee r) &&\text{Law of Impl.} \\
  &\equiv (p \vee \neg q) \vee r &&\text{Assoc.} \\
  &\equiv (\neg q \vee p) \vee r &&\text{Comm.} \\
  &\equiv \neg q \vee (p \vee r) &&\text{Assoc.} \\
  &\equiv q \rightarrow (p \vee r) &&\text{Law of Imp.}
\end{align*}
```

$$
\begin{align*}
\neg p \rightarrow (q \rightarrow r) &\equiv \neg \neg p \vee (q \rightarrow r) &&\text{Law Of Impl.} \\
&\equiv p \vee (q \rightarrow r) &&\text{Double Neg} \\
&\equiv p \vee (\neg q \vee r) &&\text{Law of Impl.} \\
&\equiv (p \vee \neg q) \vee r &&\text{Assoc.} \\
&\equiv (\neg q \vee p) \vee r &&\text{Comm.} \\
&\equiv \neg q \vee (p \vee r) &&\text{Assoc.} \\
&\equiv q \rightarrow (p \vee r) &&\text{Law of Imp.}
\end{align*}
$$

Problem 4 - Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a) $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

You may use the rule: $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

(b) $\neg p \to (q \to r) \equiv q \to (p \lor r)$

Work on part (b) with the people around you, and then we'll go over it together!

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \quad \equiv \quad \dots$

$\equiv q \rightarrow (p \vee r)$

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
	$\equiv \neg q \vee (p \vee r)$	
	$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
	$\equiv p \vee (q \rightarrow r)$	[Double Negation]
	$\equiv \neg q \vee (p \vee r)$	
	$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv \neg\neg p \vee (q \rightarrow r)$	[Law of Implication]
	$\equiv p \vee (q \rightarrow r)$	[Double Negation]
	$\equiv p \vee (\neg q \vee r)$	[Law of Implication]
	$\equiv \neg q \vee (p \vee r)$	
	$\equiv q \rightarrow (p \vee r)$	[Law of Implication]

Problem 4 - Equivalences

(b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r)$	$\equiv \neg\neg p \vee (q \to r)$	[Law of Implication]
	$\equiv p \vee (q \to r)$	[Double Negation]
	$\equiv p \vee (\neg q \vee r)$	[Law of Implication]
	$\equiv (p \vee \neg q) \vee r$	[Associativity]
	$\equiv \neg q \vee (p \vee r)$	
	$\equiv q \to (p \vee r)$	[Law of Implication]

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \lor r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv \neg\neg p \lor (q \rightarrow r)$	[Law of Implication]
	$\equiv p \lor (q \rightarrow r)$	[Double Negation]
	$\equiv p \lor (\neg q \lor r)$	[Law of Implication]
	$\equiv (p \lor \neg q) \lor r$	[Associativity]
	$\equiv (\neg q \lor p) \lor r$	[Commutativity]
	$\equiv \neg q \lor (p \lor r)$	
	$\equiv q \rightarrow (p \lor r)$	[Law of Implication]

Problem 4 - Equivalences

(b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \lor r)$

$\neg p \rightarrow (q \rightarrow r)$	$\equiv \neg\neg p \lor (q \rightarrow r)$	[Law of Implication]
	$\equiv p \lor (q \rightarrow r)$	[Double Negation]
	$\equiv p \lor (\neg q \lor r)$	[Law of Implication]
	$\equiv (p \lor \neg q) \lor r$	[Associativity]
	$\equiv (\neg q \lor p) \lor r$	[Commutativity]
	$\equiv \neg q \lor (p \lor r)$	[Associativity]
	$\equiv q \rightarrow (p \lor r)$	[Law of Implication]

Problem 4 - Equivalences

(a) $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

You may use the rule: $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$

$p \leftrightarrow q$	$\equiv (p \rightarrow q) \land (q \rightarrow p)$	[iff is two implications]
	$\equiv (\neg p \lor q) \land (q \rightarrow p)$	[Law of Implication]
	$\equiv (\neg p \lor q) \land (\neg q \lor p)$	[Law of Implication]
	$\equiv ((\neg p \lor q) \land \neg q) \lor ((\neg p \lor q) \land p)$	[Distributivity]
	$\equiv (\neg q \land (\neg p \lor q)) \lor ((\neg p \lor q) \land p)$	[Commutativity]
	$\equiv ((\neg q \land \neg p) \lor (\neg q \land q)) \lor ((\neg p \lor q) \land p)$	[Distributivity]
	$\equiv ((\neg q \land \neg p) \lor (q \land \neg q)) \lor ((\neg p \lor q) \land p)$	[Commutativity]
	$\equiv ((\neg q \land \neg p) \lor F) \lor ((\neg p \lor q) \land p)$	[Negation]
	$\equiv (\neg q \land \neg p) \lor ((\neg p \lor q) \land p)$	[Identity]
	$\equiv (\neg p \land \neg q) \lor ((\neg p \lor q) \land p)$	[Commutativity]
	$\equiv (\neg p \land \neg q) \lor (p \land (\neg p \lor q))$	[Commutativity]
	$\equiv (\neg p \land \neg q) \lor ((p \land \neg p) \lor (p \land q))$	[Distributivity]
	$\equiv (\neg p \land \neg q) \lor (F \lor (p \land q))$	[Negation]
	$\equiv (\neg p \land \neg q) \lor ((p \land q) \lor F)$	[Commutativity]
	$\equiv (\neg p \land \neg q) \lor (p \land q)$	[Identity]
	$\equiv (p \land q) \lor (\neg p \land \neg q)$	[Commutativity]

Boolean Algebra

Boolean Algebra Review

- “And” represented by ●
- “Or” represented by +
- “Not” represented by ‘

Problem 5 - Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a) $\neg p \lor (\neg q \lor (p \land q))$

(b) $\neg(p \lor (q \land p))$

Work on part (a) with the people around you, and then we'll go over it together!

Problem 5 - Boolean Algebra

(a) $\neg p \vee (\neg q \vee (p \wedge q)$

$p' + q' + pq$

Problem 5 - Boolean Algebra

(a) $\neg p \lor (\neg q \lor (p \land q))$

$p' + q' + pq$

$\equiv (pq)' + pq$ [DeMorgan's]

Problem 5 - Boolean Algebra

(a) $\neg p \lor (\neg q \lor (p \land q))$

$p' + q' + pq$

$\equiv (pq)' + pq$ [DeMorgan's]

$\equiv pq + (pq)'$ [Commutativity]

Problem 5 - Boolean Algebra

(a) $\neg p \lor (\neg q \lor (p \land q)$

$p' + q' + pq$

$\equiv (pq)' + pq$	[DeMorgan's]
$\equiv pq + (pq)'$	[Commutativity]
$\equiv 1$	[Complementarity]

Remember that 1 in Boolean Algebra is equivalent to T in propositional logic. When something always evaluates to true, it is a tautology!

Problem 5 - Boolean Algebra

(a) $\neg p \lor (\neg q \lor (p \land q))$

We can also double check our answer by simplifying the propositional logic:

$\neg p \lor (\neg q \lor (p \land q))$	$\equiv \neg p \lor ((\neg q \lor p) \land (\neg q \lor q))$	[Distributivity]
	$\equiv \neg p \lor ((\neg q \lor p) \land (q \lor \neg q))$	[Commutativity]
	$\equiv \neg p \lor ((\neg q \lor p) \land T)$	[Negation]
	$\equiv \neg p \lor (\neg q \lor p)$	[Identity]
	$\equiv (\neg q \lor p) \lor \neg p$	[Commutativity]
	$\equiv \neg q \lor (p \lor \neg p)$	[Associativity]
	$\equiv \neg q \lor T$	[Negation]
	$\equiv T$	[Domination]

Just like the Boolean Algebra expression simplified to 1, this also simplified to T!
No matter which notation we use, we end up with the same result. Yay!

Problem 5 - Boolean Algebra

(b) $\neg(p \lor (q \land p))$

$(p + qp)'$

$\equiv p'(qp)'$	[DeMorgan's]
$\equiv p'(q' + p')$	[DeMorgan's]
$\equiv p'(p' + q')$	[Commutativity]
$\equiv p'$	[Absorption]

Predicate Logic

Predicates and Quantifiers Review

- Predicate: a function that outputs true or false
  - Cat(x) := “x is a cat”
  - LessThan(x, y) := “x < y

- Work basically the same way as when we just had propositions
  - X is prime or x^2 is odd or x = 2
  - Prime(x) ∨ Odd(x^2) ∨ Equals(x, 2)

- Domain of Discourse: the types of inputs allowed in predicates
  - Numbers, mammals, cats and dogs, people in this class, etc.

- Quantifiers
  - Universal Quantifier: $\forall x$
    - For all x, for every x
  - Existential Quantifier: $\exists x$
    - There is an x, there exists an x, for some x

Problem 7 - Translate to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

(a) Every user has access to an electronic mailbox.
(b) The system mailbox can be accessed by everyone in the group if the file system is locked.
(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

Work on part (b) with the people around you, and then we'll go over it together!

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: The people in the group

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: The people in the group

Predicates:

CanAccessSM(x): x has access to the system mailbox

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: The people in the group

Predicates:
CanAccessSM(x): x has access to the system mailbox

Propositions:
p: the file system is locked

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: The people in the group

Predicates:
CanAccessSM(x): x has access to the system mailbox

Propositions:
p: the file system is locked

$$p \rightarrow \forall x \, CanAccessSM(x)$$

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: All people and mailboxes

Constant: SystemMailbox

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: All people and mailboxes

Constant: SystemMailbox

Predicates:

Access(x, y): x has access to y

InGroup(x): x is in the group

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: All people and mailboxes

Constant: SystemMailbox

Predicates:

Access(x, y): x has access to y

InGroup(x): x is in the group

Propositions:

p: the file system is locked

Problem 7 - Translate to Logic

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Domain: All people and mailboxes

Constant: SystemMailbox

Predicates:

Access(x, y): x has access to y

InGroup(x): x is in the group

Propositions:

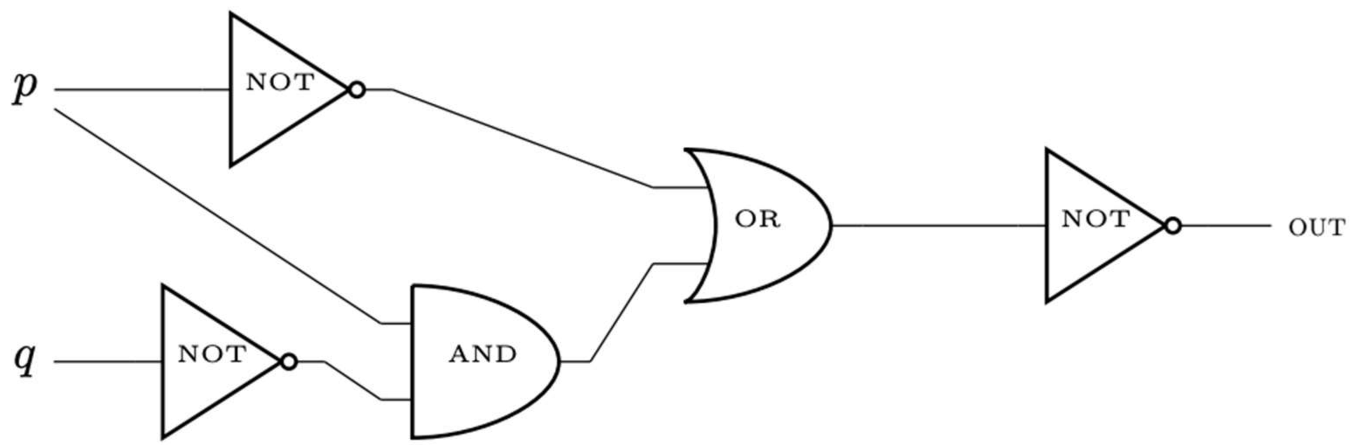
p: the file system is locked

$$p \rightarrow \forall x\ (\text{InGroup}(x) \rightarrow \text{Access}(x, \text{SystemMailbox}))$$

Circuit Logic

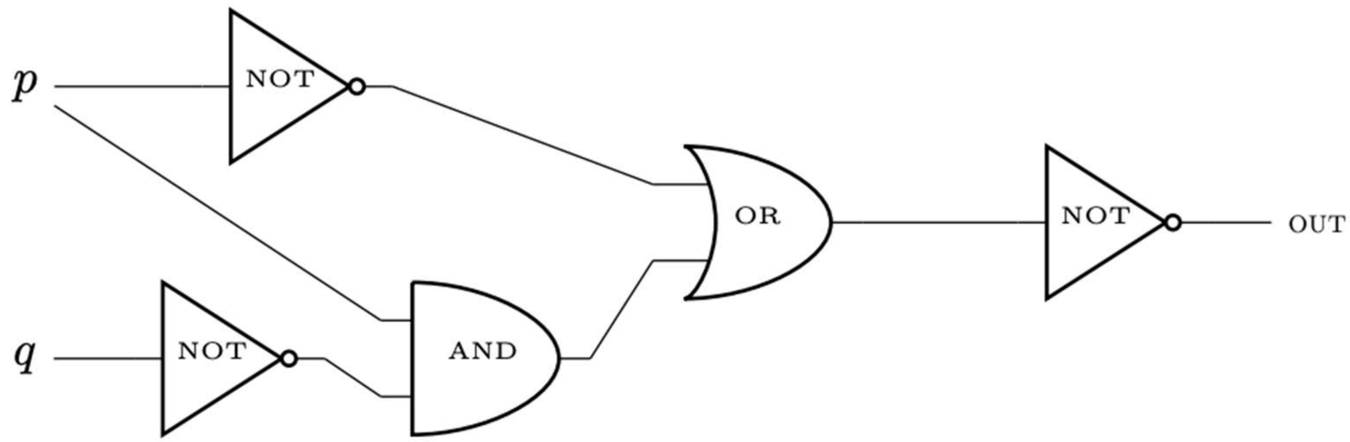
Problem 3 - Circuitous

Translate the following circuit into a logical expression.



Work on the problem with the people around you, and then we'll go over it together!

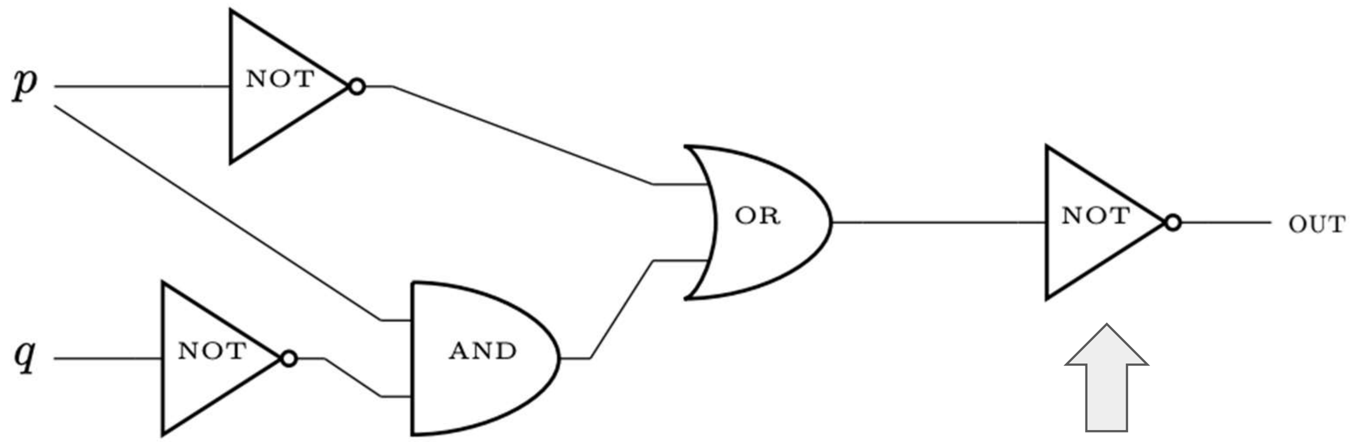
Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee (p \wedge \neg q))$

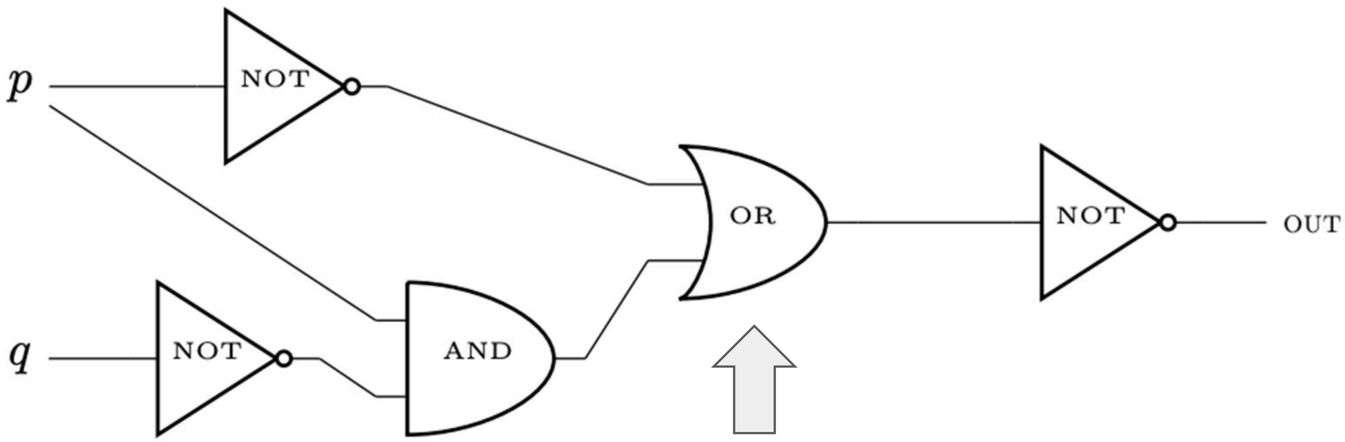
Problem 3 - Circuitous



Solution:

$\neg(\quad)$

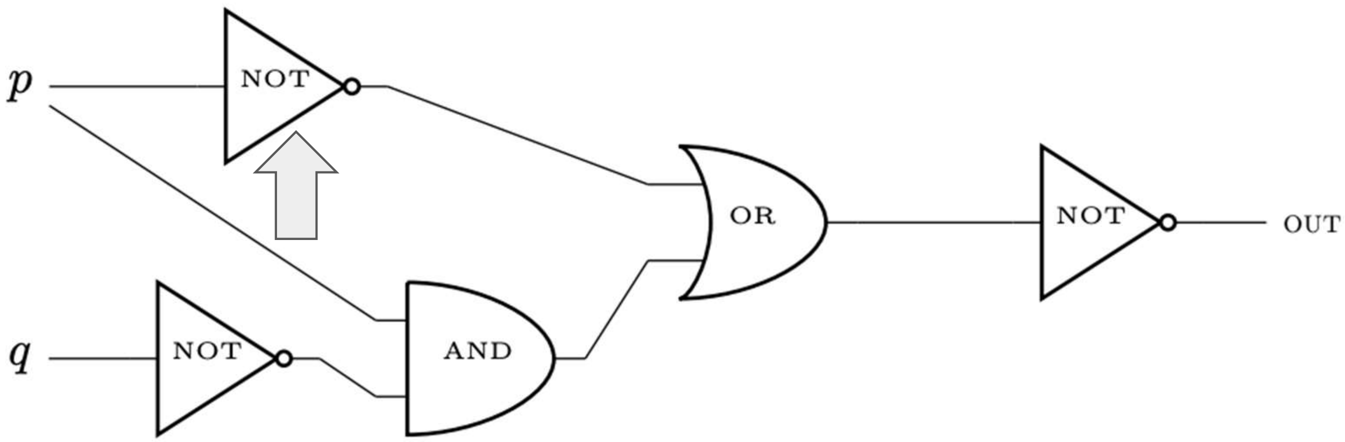
Problem 3 - Circuitous



Solution:

$\neg(\quad \vee \quad (\quad\quad))$

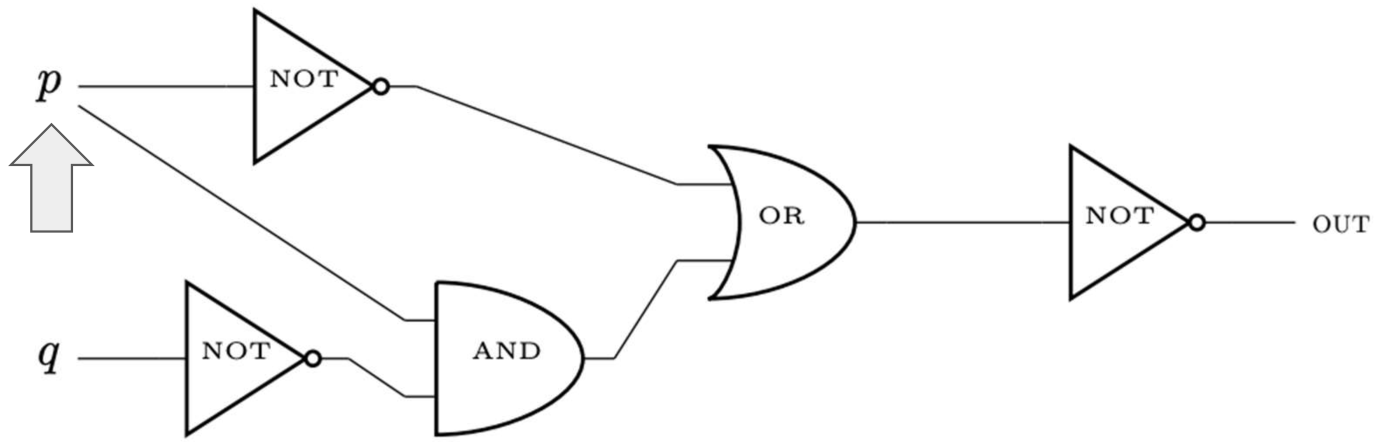
Problem 3 - Circuitous



Solution:

$\neg(\neg\ \vee\ (\ \ ))$

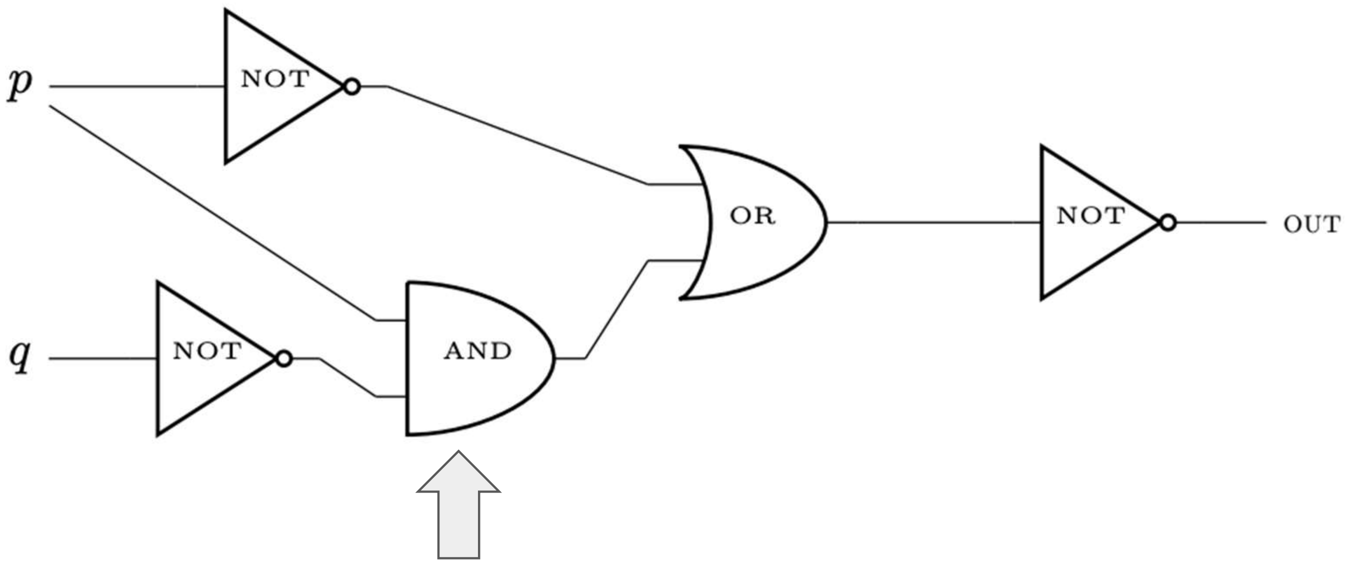
Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee \boxed{\quad}))$

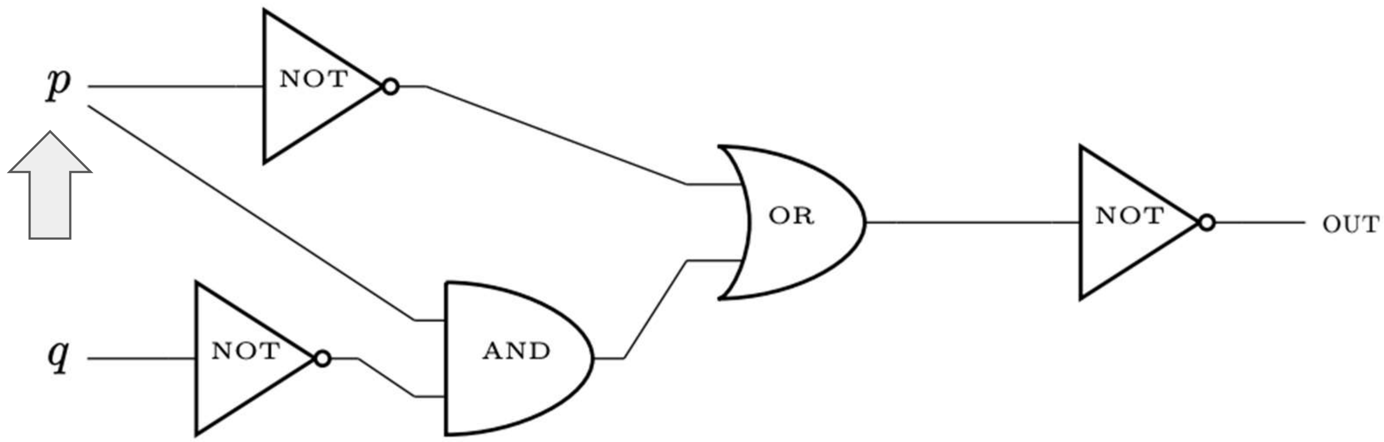
Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee (\square \wedge \square))$

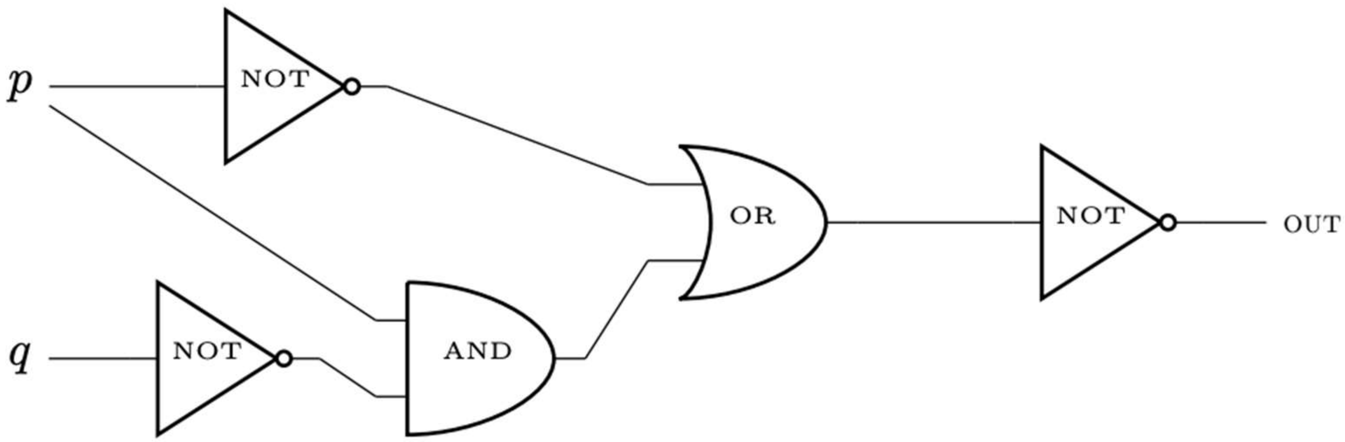
Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee (p \wedge \square))$

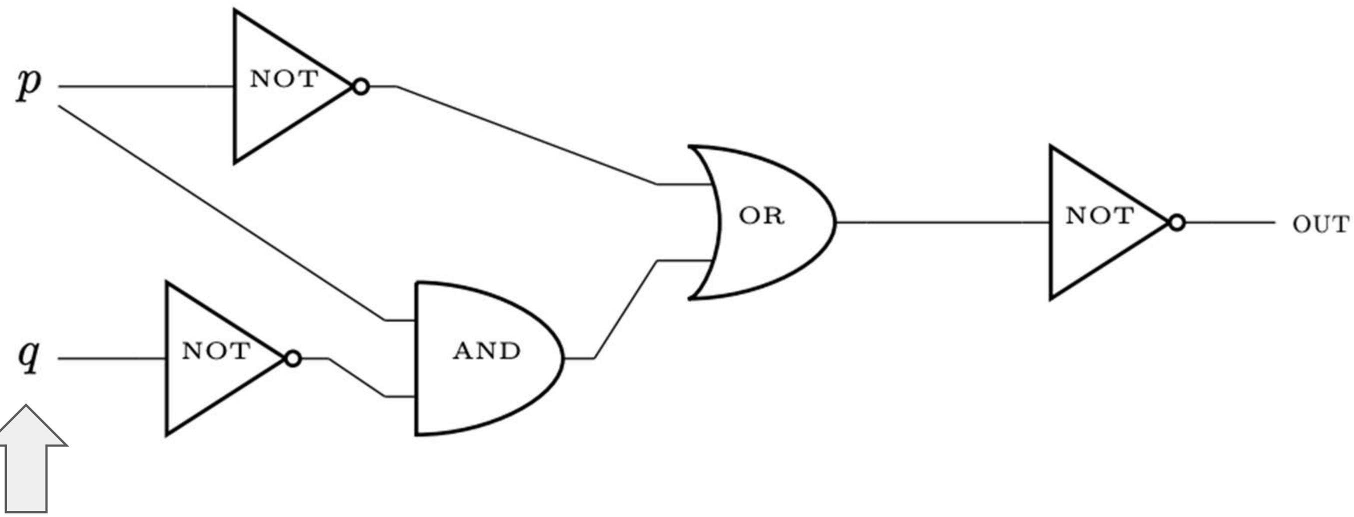
Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee (p \wedge \neg \square))$

Problem 3 - Circuitous



Solution:

$\neg(\neg p \vee (p \wedge \neg q))$

That’s All, Folks!

Any questions?