Law of Implication

Implications are hard.
AND/OR/NOT make more intuitive sense to me…
can we rewrite implications using just ANDs ORs and NOTs?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

One approach: think "when is this implication false?" then negate it (you might want one of DeMorgan's Laws!

Converse, Contrapositive

Implication:

If it's raining, then I have my umbrella. $p \to q$

Contrapositive:

$\neg q \to \neg p$ If I don't have my umbrella, then it is not raining.

Converse:

If I have my umbrella, then it is raining. $q \to p$

Inverse:

$\neg p \to \neg q$ If it is not raining, then I don't have my umbrella.

How do these relate to each other?

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T						
T	F						
F	T						
F	F						

Properties of Logical Connectives

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Our First Proof

$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$

None of the rules look like this

Practice of Proof-Writing:
Big Picture…WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines…maybe the "$\neg a$" $\equiv (\neg a \vee b)$ came from there? Maybe that simplifies down to $\neg a$