Our First Proof

$(a \wedge b) \vee (\neg a \wedge b) \vee (\neg a \wedge \neg b) \equiv$

None of the rules look like this

Practice of Proof-Writing:
Big Picture…WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines…maybe the "$\neg a$" $\equiv (\neg a \vee b)$ came from there? Maybe that simplifies down to $\neg a$

Simplify $\mathrm{T} \wedge (\neg a \vee b)$ to $(\neg a \vee b)$

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Vocabulary!

A proposition is a….

Tautology **if it is always true.**
Contradiction **if it is always false.**
Contingency **if it can be both true and false.**

$p \vee \neg p$

Tautology

If p is true, $p \vee \neg p$ is true; if p is false, $p \vee \neg p$ is true.

$p \oplus p$

Contradiction

If p is true, $p \oplus p$ is false; if p is false, $p \oplus p$ is false.

$(p \to q) \wedge p$

Contingency If p is true and q is true, $(p \to q) \wedge p$ is true;
If p is true and q is false, $(p \to q) \wedge p$ is false.

Meet Boolean Algebra

Name	Variables	"True/False"	"And"	"Or"	"Not"	Implication
Java Code	`boolean b`	`true,false`	`&&`	`||`	`!`	No special symbol
Propositional Logic	"p, q, r"	`T, F`	$\wedge$	$\vee$	$\neg$	$\to$
Circuits	Wires	1, 0	And	Or	Not	No special symbol
Boolean Algebra	a, b, c	1,0	$\cdot$ ("multiplication")	$+$ ("addition")	$'$ (apostrophe after variable)	No special symbol

Propositional logic

$(p \wedge q \wedge r) \vee s \vee \neg t$

Boolean Algebra

$pqr + s + t'$