A proof!

What's the analogue of DeMorgan's Laws…

$\bar{A} \cap \bar{B} = \overline{A \cup B}$

$A = B \equiv \forall x(x \in A \leftrightarrow x \in B) \equiv A \subseteq B \land B \subseteq A$

$\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$
Let x be an arbitrary element of $\bar{A} \cap \bar{B}$.
By definition of $\cap$ $x \in \bar{A}$ and $x \in \bar{B}$. By definition of complement, $x \notin A \land x \notin B$.
Applying DeMorgan's Law, we get $\neg(x \in A \lor x \in B)$.
That is, x is in the complement of the set that contains all x such that $x \in A \lor x \in B$.
So, by definition of union x $\in \overline{A \cup B}$, as required.
Since x was arbitrary $\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$

$\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$
Let x be an arbitrary element of $\overline{A \cup B}$.
By definition of complement, x is not an element of $A \cup B$. Applying the definition of union, we get, $\neg(x \in A \lor x \in B)$
Applying DeMorgan's Law, we get: $x \notin A \land x \notin B$
By definition of complement, $x \in \bar{A} \land x \in \bar{B}$. So by definition of intersection, we get $x \in \bar{A} \cap \bar{B}$
Since x was arbitrary $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$

Since the subset relation holds in both directions, we have $\bar{A} \cap \bar{B} = \overline{A \cup B}$

Two claims, two proof techniques

Suppose I claim that for all sets A, B, C: $A \cap B \subseteq C$

That…doesn't look right.

How do you prove me wrong?

What am I trying to prove? First write symbols for "¬(for all sets A, B, C …)"

Then 'distribute' the negation sign.

Proof By Cases

Let $A = \{x : \texttt{Prime}(x)\}$, $B = \{x: \texttt{Odd}(x) \vee \texttt{PowerOfTwo}(x)\}$

Where $\texttt{PowerOfTwo}(x) \coloneqq \exists c(\texttt{Integer}(c) \wedge x = 2\text{^}c)$

Prove $A \subseteq B$

Divides

Divides

For integers x, y we say $x|y$ ("x divides y") iff there is an integer z such that $xz = y$.

Which of these are true?

$2\|4$	$4\|2$	$2\| -2$
$5\|0$	$0\|5$	$1\|5$