More Induction

Induction doesn't **only** work for code!

Show that $\sum_{i=0}^{n} 2^i = 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$.

Let $P(n)$ be "$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$."

We show $P(n)$ holds for all n by induction on n.

Base Case ( )

Inductive Hypothesis:

Inductive Step:

$P(n)$ holds for all $n \geq 0$ by the principle of induction.

Let's Try Another Induction Proof

Let $g(n) = \begin{cases} 2 & \text{if } n = 2 \\ g(n-1)^2 + 3g(n-1) & \text{if } n > 2 \end{cases}$

Prove $g(n)$ is even for all $n \geq 2$ by induction on n.

Let's just set this one up, we'll leave the individual pieces as exercises.

Induction on Primes.

Let $P(n)$ be "n can be written as a product of primes."

We show $P(n)$ for all $n \geq 2$ by induction on n.

Base Case ($\boldsymbol{n = 2}$): 2 is a product of just itself. Since 2 is prime, it is written as a product of primes.

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary integer $k \geq 2$.

Inductive Step:
Case 1, $k + 1$ is prime: then $k + 1$ is automatically written as a product of primes.

Case 2, $k + 1$ is composite:

Therefore $P(k + 1)$.

$P(n)$ holds for all $n \geq 2$ by the principle of induction.

Making Induction Proofs Pretty

All of our strong induction proofs will come in 5 easy(?) steps!

1. Define $P(n)$. State that your proof is by induction on n.
2. Base Case: Show $P(b)$ i.e. show the base case
3. Inductive Hypothesis: Suppose $\mathrm{P(b)} \wedge \cdots \wedge P(k)$ for an arbitrary $k \geq b$.
4. Inductive Step: Show $P(k + 1)$ (i.e. get $[\mathrm{P(b)} \wedge \cdots \wedge P(k)] \rightarrow P(k + 1)$)
5. Conclude by saying $P(n)$ is true for all $n \geq b$ by the principle of induction.