Recursive Definitions of Sets

Q1: What is this set?

Basis Step: $6 \in S, 15 \in S$

Recursive Step: If $x, y \in S$ then $x + y \in S$

Q2: Write a recursive definition for the set of powers of 3 $\{1,3,9,27, \ldots\}$

Basis Step:

Recursive Step:

Structural Induction

Let $P(x)$ be "x is divisible by 3"

We show $P(x)$ holds for all $x \in S$ by structural induction.

Base Cases:
Inductive Hypothesis:

Inductive Step:

We conclude $P(x) \forall x \in \mathrm{S}$ by the principle of induction.

Basis: $6 \in \mathrm{S}, 15 \in S$
Recursive: if $x, y \in S$ then $x + y \in S$.

Structural Induction Template

1. Define $P()$ Show that $P(x)$ holds for all $x \in S$. State your proof is by structural induction.

2. Base Case: Show $P(x)$
[Do that for every base cases x in S.]

Let y be an arbitrary element of S not covered by the base cases. By the exclusion rule, $y =$<recursive rules>

3. Inductive Hypothesis: Suppose $P(x)$
[Do that for every x listed as in S in the recursive rules.]

4. Inductive Step: Show $P()$ holds for y.
[You will need a separate case/step for every recursive rule.]

5. Therefore $P(x)$ holds for all $x \in S$ by the principle of induction.

Claim for all $x, y \in \Sigma^*$ len(x·y)=len(x) + len(y).

Define Let $P(y)$ be "len(x·y)=len(x) + len(y) for all $x \in \Sigma^*$."

We prove $P(y)$ for all $y \in \Sigma^*$ by structural induction.

Base Case:

Inductive Hypothesis:

Inductive Step:

len(ε)=0;
len(wa)=len(w)+1 for $w \in \Sigma^*$, $a \in \Sigma$
Σ^*:Basis: $\varepsilon \in \Sigma^*$.
Recursive: If $w \in \Sigma^*$ and $a \in \Sigma$ then $wa \in \Sigma^*$