

What do real numbers look like

0. 3 3 3 3 3 3 3 3 3...

0. 2 7 2 7 2 8 5 4...

0. 1 4 1 5 9 2 6 5...

0. 2 2 2 2 2 2 2 2 2...

0. 1 2 3 4 5 6 7 8...

0. 9 8 7 6 5 4 3 2...

0. 8 2 7 6 4 5 7 4...

0. 5 9 4 2 7 5 1 7...

A string of digits!

Well not a "string" An infinitely long sequence of digits is more accurate.

Proof that $[0,1)$ is not countable

Suppose, for the sake of contradiction, that there is a list of them:

| Number | Digits after decimal | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ... |
|--------|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $f(0)$ | 0. | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | ... |
| $f(1)$ | 0. | 2 | 7 | 2 | 7 | 2 | 8 | 5 | 4 | ... |
| $f(2)$ | 0. | 1 | 4 | 1 | 5 | 9 | 2 | 6 | 5 | ... |
| $f(3)$ | 0. | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | ... |
| $f(4)$ | 0. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... |
| $f(5)$ | 0. | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | ... |
| $f(6)$ | 0. | 8 | 2 | 7 | 6 | 4 | 5 | 7 | 4 | ... |
| $f(7)$ | 0. | 5 | 9 | 4 | 2 | 7 | 5 | 1 | 7 | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |

The Halting Problem

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Given: source code for a program P and x an input we could give to P
Return: True if P will halt on x , False if it runs forever (e.g. goes in an infinite loop or infinitely recurses)

This would be super useful to solve!

We can't solve it...let's find out why.

A Very Tricky Program.

```
Diagonal.java(String x){
    Run H.exe on input <x, x>
    if(H.exe says "x halts on x")
        while(true){//Go into an infinite loop
            int x=2+2;
        }
    else //H.exe says "x doesn't halt on x"
        return; //halt.
}
```