Problem 4- Section 04

Write an English proof, proving the following set identity

Let the universal set be $\boldsymbol{U}$. Prove $A \cap B' \subseteq A \backslash B$ for any sets A, B.

Work on this problem with the people around you.

(1) Translate the claim to predicate
(2) Write out the skeleton

Let's Write it Out:

$\forall x(x \in A \cap B' \rightarrow x \in A \backslash B)$

Remember our Strong Induction Template!

Let $P(n)$ be "(whatever you're trying to prove)".
We show $P(n)$ holds for all $n \geq b_{min}$ by induction on n.

Base Case: Show $P(b_{min}), P(b_{min+1}), \ldots, P(b_{max})$ are all true.

Inductive Hypothesis: Suppose $P(b_{min}) \wedge \cdots \wedge P(k)$ hold for an arbitrary $k \geq b_{max}$.

Inductive Step: Show $P(k+1)$ (i.e. get $P(b_{min}) \wedge \cdots \wedge P(k) \rightarrow P(k+1)$)

Conclusion: Therefore, $P(n)$ holds for all $n \geq b_{min}$ by the principle of induction.

Let's Write it Out:

Consider the function $a(n)$ defined for $n \geqslant 1$ recursively as follows.

$$a(1) = 1$$

$$a(2) = 3$$

$$a(n) = 2a(n-1) - a(n-2) \text{ for } n \geqslant 3$$

Use strong induction to prove that $a(n) = 2n - 1$ for all $n \geqslant 1$.