CSE 311 Section MR

Midterm Review

Administrivia

Announcements & Reminders

- HW5 (BOTH PARTS)
  - BOTH PARTS were due Wednesday 11/8 @ 10pm
  - Late due date Friday 11/10

- Midterm is Coming Next Week!!!
  - Wednesday 10/15 @ 6-7:30 pm in BAG 131 and 154
  - If you cannot make it, please let us know ASAP and we will schedule you for a makeup

Problem 1: Translation

Problem 1 – Translation

Let your domain of discourse be all coffee drinks. You should use the following predicates:

- soy(x) is true iff x contains soy milk.
- whole(x) is true iff x contains whole milk.
- sugar(x) is true iff x contains sugar
- decaf(x) is true iff x is not caffeinated.
- vegan(x) is true iff x is vegan.
- RobbieLikes(x) is true iff Robbie likes the drink x.

Translate each of the following statements into predicate logic. You may use quantifiers, the predicates above, and usual math connectors like $=$ and $\neq$.

a) Coffee drinks with whole milk are not vegan

b) Robbie only likes one coffee drink, and that drink is not vegan

c) There is a drink that has both sugar and soy milk.

Work on this problem with the people around you.

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Translate the following symbolic logic statement into a (natural) English sentence. Take advantage of domain restriction.

$$\forall x([\text{decaf}(x) \land \text{RobbieLikes}(x)] \rightarrow \text{sugar}(x))$$

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Problem 2: Set Theory

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$$\forall X \left[\Big((A \subseteq B) \wedge \big(X \in \mathcal{P}(A) \big) \Big) \rightarrow \big(X \in \mathcal{P}(B) \big) \right]$$

Then, write the proof.

Work on this problem with the people around you.

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$$\forall X\left[\left((A \subseteq B) \wedge \left(X \in \mathcal{P}(A)\right)\right) \rightarrow \left(X \in \mathcal{P}(B)\right)\right]$$

Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Problem 3: Number Theory

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Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$.

a) Show that if an integer y satisfies $y \equiv 1 \pmod{p}$, then $y^2 \equiv 1 \pmod{p}$.

b) Repeat part (a), but don't use any theorems from the Number Theory Reference Sheet. That is, show the claim directly from the definitions.

c) From part (a), we can see that $x \% p$ can equal 1. Show that for any integer x, if $x^2 \equiv 1 \pmod{p}$, then $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$. That is, show that the only value $x \% p$ can take other than 1 is $p - 1$.
Hint: Suppose you have an x such that $x^2 \equiv 1 \pmod{p}$ and use the fact that $x^2 - 1 = (x - 1)(x + 1)$
Hint: You may the following theorem without proof: if p is prime and $p \mid (ab)$ then $p \mid a$ or $p \mid b$.

Work on this problem with the people around you.

Problem 3 – Number Theory

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b) Repeat part (a), but don't use any theorems from the Number Theory Reference Sheet. That is, show the claim directly from the definitions.

Problem 3 – Number Theory

Let p be a prime number at least 3 and let x be an integer such that $x^2 \% p = 1$

c) From part (a), we can see that $x\%p$ can equal 1. Show that for any integer x, if $x^2 \equiv 1 \pmod{p}$, then $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$. That is, show that the only value $x\%p$ can take other than 1 is $p - 1$.
Hint: Suppose you have an x such that $x^2 \equiv 1 \pmod{p}$ and use the fact that $x^2 - 1 = (x - 1)(x + 1)$
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Problem 4: Induction

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For any $n \in \mathbb{N}$, define S_n to be the sum of the squares of the first n positive integers, or $S_n = 1^2 + 2^2 + \cdots + n^2$.

Prove that for all $n \in \mathbb{N}$, $S_n = \frac{1}{6}n(n+1)(2n+1)$.

Work on this problem with the people around you.

Problem 4 – Induction

$S_n = 1^2 + 2^2 + \cdots + n^2$.

Prove that for all $n \in \mathbb{N}$, $S_n = \frac{1}{6}n(n+1)(2n+1)$.

Let $P(n)$ be "". We show $P(n)$ holds for (some) n by induction on n.

Base Case: $P(b)$:

Inductive Hypothesis: Suppose $P(k)$ holds for an arbitrary $k \geq b$.

Inductive Step: Goal: Show $P(k+1)$:

Conclusion: Therefore, $P(n)$ holds for (some) n by the principle of induction.

Problem 5: Strong Induction

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Robbie is planning to buy snacks for the members of his competitive roller-skating troupe. However, his local grocery store sells snacks in packs of 5 and packs of 7.

Prove that Robbie can buy exactly n snacks for all integers $n \geq 24$

Work on this problem with the people around you.

Problem 5 – Strong Induction

Can buy snacks in packs of 5 and packs of 7. Prove that Robbie can buy exactly n snacks for all integers $n \geq 24$

Let $P(n)$ be "".

We show $P(n)$ holds for all $n \geq b_{min}$ by strong induction on n.

<u>Base Cases:</u>

<u>Inductive Hypothesis:</u> Suppose $P(b_{min}) \wedge \cdots \wedge P(k)$ hold for an arbitrary all $k \geq b_{max}$.

<u>Inductive Step:</u> Goal: Show $P(k+1)$:

<u>Conclusion:</u> Therefore, $P(n)$ holds for all $n \geq b_{min}$ by the principle of induction.

That’s All, Folks!

Thanks for coming to section this week!
Any questions?