Quiz Section 2: Circuits and Predicate Logic

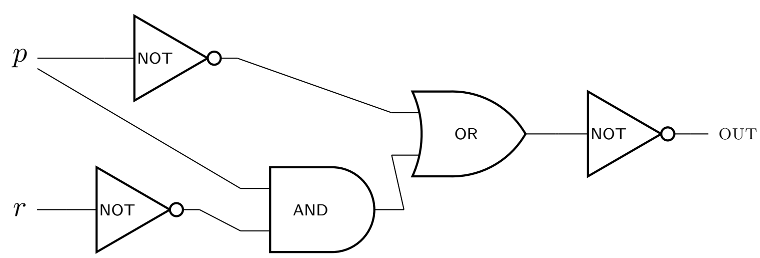
Review

Boolean Algebra

Closure	$a + b$ is in $\mathbb{B}$	$a \bullet b$ is in $\mathbb{B}$
Commutativity	$a + b = b + a$	$a \bullet b = b \bullet a$
Associativity	$a + (b + c) = (a + b) + c$	$a \bullet (b \bullet c) = (a \bullet b) \bullet c$
Identity	$a + 0 = a$	$a \bullet 1 = a$
Distributivity	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
Complementarity	$a + a' = 1$	$a \bullet a' = 0$
Null	$a + 1 = 1$	$a \bullet 0 = 0$
Idempotency	$a + a = a$	$a \bullet a = a$
Involution	$(a')' = a$	
DeMorgan	$(a + b + \cdots)' = a' \bullet b' \bullet \cdots$	$(a \bullet b \bullet \cdots)' = a' + b' + \cdots$
Uniting	$a \bullet b + a \bullet b' = a$	$(a + b) \bullet (a + b') = a$
Absorption	$a + a \bullet b = a$	$a \bullet (a + b) = a$

Task 1 – Circuitous

Translate the following circuit into a logical expression.



Task 2 – More Circuits

Let Q be defined by $Q(p, q) = (\neg p) \oplus q$. Using only NOT, OR and Q gates, draw a circuit that represents the logical expression $(a \wedge b) \oplus c$.

Task 3 – Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

a) $\neg p \vee (\neg q \vee (p \wedge q))$

b) $\neg (p \vee (q \wedge p))$

Task 4 – Canonical Forms

Consider the boolean functions $F(A, B, C)$ and $G(A, B, C)$ specified by the following truth table:

A	B	C	$F(A,B,C)$	$G(A,B,C)$
1	1	1	1	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

a) Write the DNF and CNF expressions for $F(A, B, C)$.

b) Write the DNF and CNF expressions for $G(A, B, C)$.

c) Simplify your CNF form for $G(A, B, C)$.

Task 5 – Translate to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates.

a) Every user has access to an electronic mailbox.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

Task 6 – Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers and all print jobs.

a) $\exists p\ (F(p) \land B(p)) \rightarrow \exists j\ L(j)$

b) $(\forall j\ B(j)) \rightarrow (\exists p\ Q(p))$

c) $\exists j\ (Q(j) \land L(j)) \rightarrow \exists p\ F(p)$

d) $(\forall p\ B(p) \land \forall j\ Q(j)) \rightarrow \exists j\ L(j)$

Task 7 – Domain Restriction

Translate each of the following sentences into logical notation. These translations require some of our quantifier tricks. You may use the operators $+$ and $\cdot$ which take two numbers as input and evaluate to their sum or product, respectively. Remember:

- To restrict the domain under a $\forall$ quantifier, add a hypothesis to an implication.
- To restrict the domain under an $\exists$ quantifier, AND in the restriction.
- If you want variables to be different, you have to explicitly require them to be not equal.

a) Domain: Positive integers; Predicates: $\mathtt{Even}, \mathtt{Prime}, \mathtt{Equal}$
"There is only one positive integer that is prime and even."

b) Domain: Real numbers; Predicates: $\mathtt{Even}, \mathtt{Prime}, \mathtt{Equal}$
"There are two different prime numbers that sum to an even number."

c) Domain: Real numbers; Predicates: $\mathtt{Even}, \mathtt{Prime}, \mathtt{Equal}$
"The product of two distinct prime numbers is not prime."

d) Domain: Real numbers; Predicates: $\mathtt{Even}, \mathtt{Prime}, \mathtt{Equal}, \mathtt{Postivite}, \mathtt{Greater}, \mathtt{Integer}$
"For every positive integer, there is a greater even integer"