Law of Implication

Implications are hard.
AND/OR/NOT make more intuitive sense to me…
can we rewrite implications using just ANDs ORs and NOTs?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

One approach: think "when is this implication false?" then negate it (you might want one of DeMorgan's Laws!

Converse, Contrapositive

Implication:

If it's raining, then I have my umbrella. $p \to q$

Contrapositive:

$\neg q \to \neg p$ If I don't have my umbrella, then it is not raining.

Converse:

If I have my umbrella, then it is raining. $q \to p$

Inverse:

$\neg p \to \neg q$ If it is not raining, then I don't have my umbrella.

How do these relate to each other?

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T						
T	F						
F	T						
F	F						

Properties of Logical Connectives

These identities hold for all propositions p, q, r

- Identity
  - $p \land \mathrm{T} \equiv p$
  - $p \lor \mathrm{F} \equiv p$
- Domination
  - $p \lor \mathrm{T} \equiv \mathrm{T}$
  - $p \land \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \lor p \equiv p$
  - $p \land p \equiv p$
- Commutative
  - $p \lor q \equiv q \lor p$
  - $p \land q \equiv q \land p$
- Associative
  - $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  - $(p \land q) \land r \equiv p \land (q \land r)$
- Distributive
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
- Absorption
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$
- Negation
  - $p \lor \neg p \equiv \mathrm{T}$
  - $p \land \neg p \equiv \mathrm{F}$

Our First Proof

$(a \land b) \lor (\neg a \land b) \lor (\neg a \land \neg b) \equiv$

None of the rules look like this

Practice of Proof-Writing:
Big Picture…WHY do we think this might be true?

The last two "pieces" came from the vacuous proof lines…maybe the "$\neg a$" $\equiv (\neg a \lor b)$ came from there? Maybe that simplifies down to $\neg a$