CSE 311 Section 4

English Proofs & Set Theory

Administrivia

Announcements & Reminders

- HW2
    - If you think something was graded incorrectly, submit a regrade request!
- HW3 was due yesterday 1/24 @ 11:59PM on Gradescope
    - Use late days if you need them!
- HW4
    - Due Friday 1/31 @ 11:59pm

References

- Helpful reference sheets can be found on the course website!
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/
- How to LaTeX (found on Assignments page of website):
  - https://courses.cs.washington.edu/courses/cse311/23wi/assignments/HowToLaTeX.pdf
- Set Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/reference-sets.pdf
- Number Theory Reference Sheet
  - https://courses.cs.washington.edu/courses/cse311/23wi/resources/reference-number-theory.pdf
- Plus more!

English Proofs

Writing a Proof (symbolically or in English)

- Don't just jump right in!

1. Look at the **claim**, and make sure you know:
   - What every word in the claim means
   - What the claim as a whole means
2. Translate the claim in predicate logic.
3. Next, write down the **Proof Skeleton**:
   - Where to **start**
   - What your **target** is
4. Then once you know what claim you are proving and your starting point and ending point, you can finally write the proof!

Helpful Tips for English Proofs

- Start by introducing your assumptions
  - Introduce variables with "let"
    - "Let x be an arbitrary prime number…"
  - Introduce assumptions with "suppose"
    - "Suppose that $y \in A \land y \notin B$…"
- When you supply a value for an existence proof, use "Consider"
  - "Consider $x = 2$…"
- **ALWAYS** state what type your variable is (integer, set, etc.)
- Universal Quantifier means variable must be arbitrary
- Existential Quantifier means variable can be specific

Divisibility

Problem 1

(a) Identify the statements that are true for divides

(i) 1 | 3

(ii) 3 | 1

(iii) 2 | 2018

(iv) -2 | 12

(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

Problem 1

(a) Identify the statements that are true for divides

(i) 1 | 3

(ii) 3 | 1

(iii) 2 | 2018

(iv) -2 | 12

(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

i. True: 3 = 1 * 3

Problem 1

(a) Identify the statements that are true for divides

(i) 1 | 3

(ii) 3 | 1

(iii) 2 | 2018

(iv) -2 | 12

(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

i. True: 3 = 1 * 3

ii. False: $1 \neq 3 * k$

Problem 1

(a) Identify the statements that are true for divides

(i) 1 | 3

(ii) 3 | 1

(iii) 2 | 2018

(iv) -2 | 12

(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

i. True: 3 = 1 * 3
ii. False: 1 ≠ 3 * k
iii. True: 2018 = 2 * 1009

Problem 1

(a) Identify the statements that are true for divides

(i) 1 | 3

(ii) 3 | 1

(iii) 2 | 2018

(iv) -2 | 12

(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

i. True: 3 = 1 * 3
ii. False: 1 ≠ 3 * k
iii. True: 2018 = 2 * 1009
iv. True: 12 = -2 * 6

Problem 1

(a) Identify the statements that are true for divides

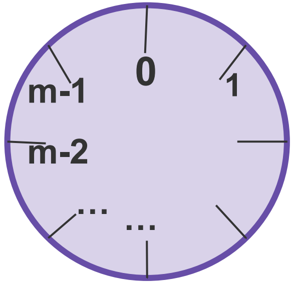
(i) 1 | 3
(ii) 3 | 1
(iii) 2 | 2018
(iv) -2 | 12
(v) 1 * 2 * 3 * 4 | 1 * 2 * 3 * 4 * 5

i. True: 3 = 1 * 3
ii. False: 1 ≠ 3 * k
iii. True: 2018 = 2 * 1009
iv. True: 12 = -2 * 6
v. True: 5! = 4! * 5

Mod

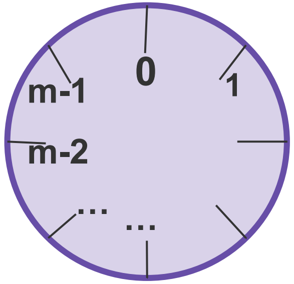
a ≡ b (mod m)

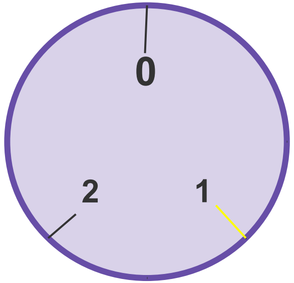
Imagine a clock with m numbers



a ≡ b (mod m)

Imagine a clock with m numbers





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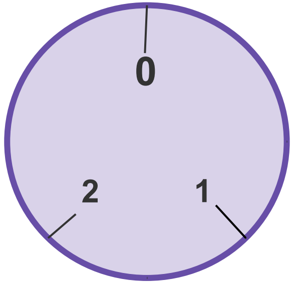
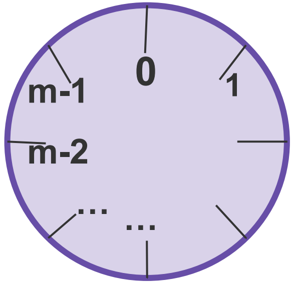
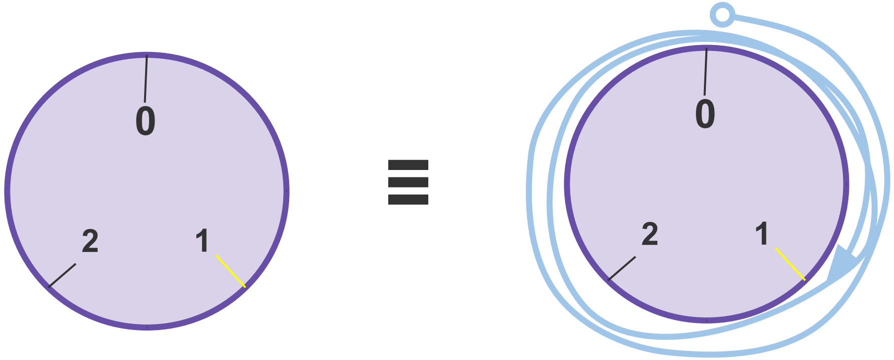


1 (mod 3) **VS** 10 (mod 3)

a ≡ b (mod m)

Imagine a clock with m numbers





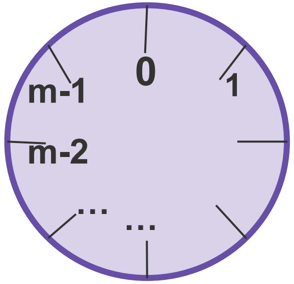
1 (mod 3)

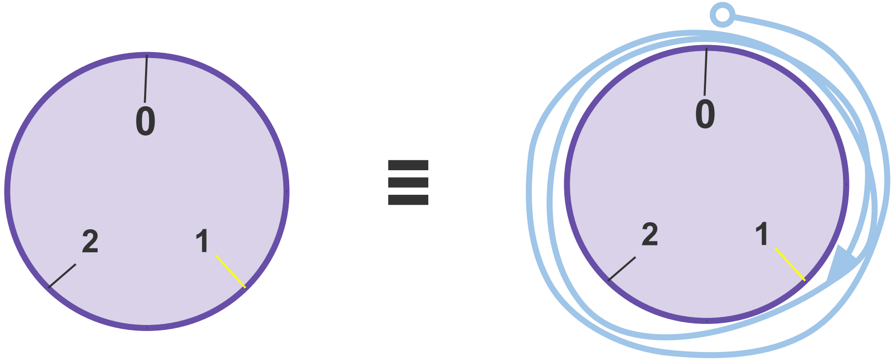
VS

10 (mod 3)

a ≡ b (mod m)

Imagine a clock with m numbers



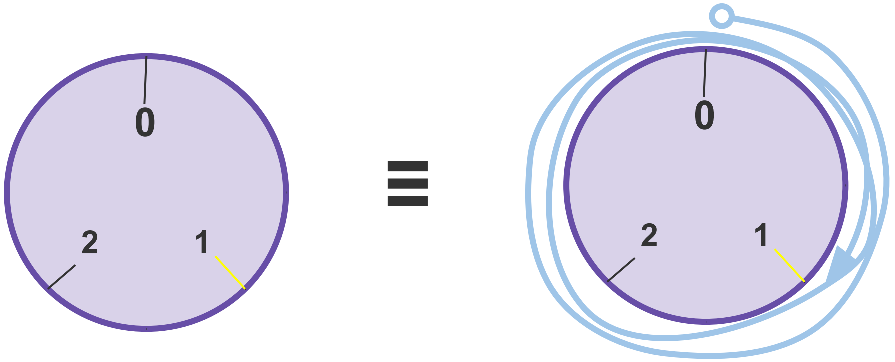


1 (mod 3) **VS** 10 (mod 3)

So we can say that **a ≡ b (mod m)** where a and b are in the same position in the mod clock

Divides

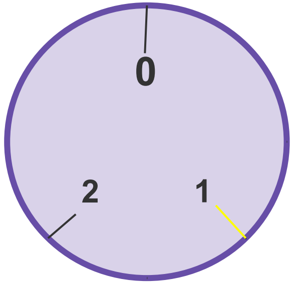
What if we "unroll" this clock?



1 (mod 3) **VS** 10 (mod 3)

Divides

What if we “unroll” this clock?



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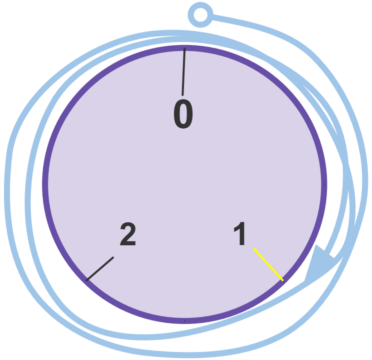
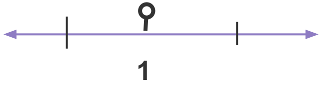
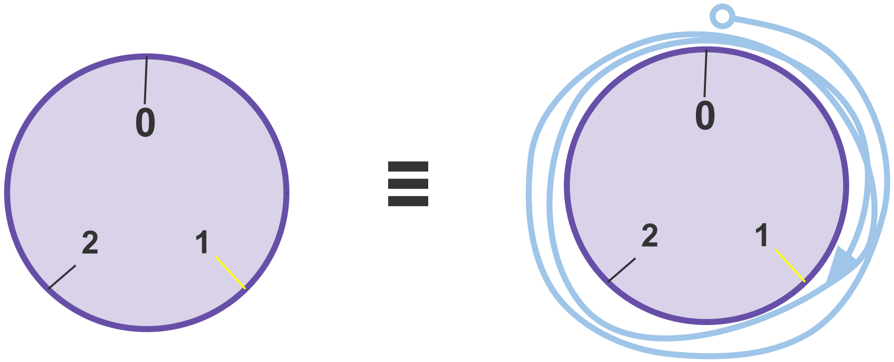


1 (mod 3) **VS** 10 (mod 3)

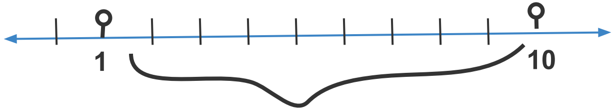


Divides

What if we “unroll” this clock?



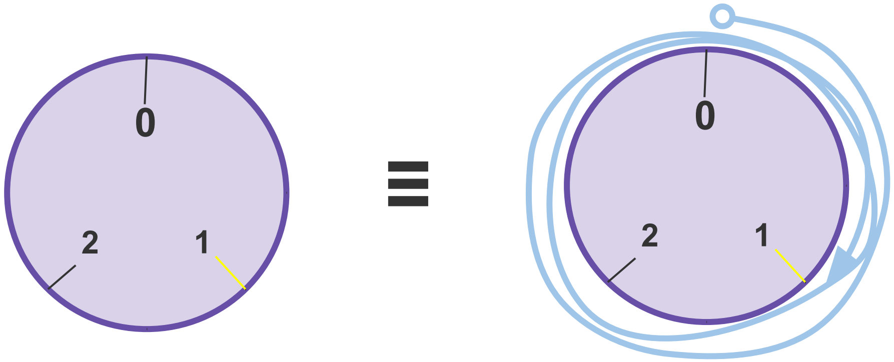
1 (mod 3) **VS** 10 (mod 3)



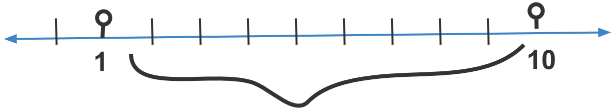
Anything interesting?

Divides

What if we "unroll" this clock?



1 (mod 3) **VS** 10 (mod 3)



(10-1) = 9

9 ÷ 3 = 3 so 3 | 9

Anything interesting?

$3 \nmid 10$ and $3 \nmid 1$ BUT $3 | 9$

So m divides the <u>difference</u> between a and b

Formalizing Mod and Divides

Equivalence in modular arithmetic

Let $a \in \mathbb{Z}, b \in \mathbb{Z}, n \in \mathbb{Z}$ and $n > 0$.

We say $a \equiv b \ (mod\ n)$ if and only if $n | (b - a)$

n | (b-a)

Problem 1

(b) Identify the statements that are true for mod using the equivalence definition!

(i) $-3 \equiv 3 \pmod{3}$

(ii) $0 \equiv 9000 \pmod{9}$

(iii) $44 \equiv 13 \pmod{7}$

(iv) $-58 \equiv 707 \pmod{5}$

(v) $58 \equiv 707 \pmod{5}$

Problem 1

(b) Identify the statements that are true for mod using the equivalence definition!

(i) $-3 \equiv 3 \pmod 3$

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(v) $58 \equiv 707 \pmod 5$

i. True: $3|(3+3) = 3|6$

Problem 1

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(iii) $44 \equiv 13 \pmod{7}$

(iv) $-58 \equiv 707 \pmod{5}$

(v) $58 \equiv 707 \pmod{5}$

i. True: $3|(3+3) = 3|6$

ii. True: $9|(9000-0) = 9|9000$

Problem 1

(b) Identify the statements that are true for mod using the equivalence definition!

(i) $-3 \equiv 3 \pmod 3$

(ii) $0 \equiv 9000 \pmod 9$

(iii) $44 \equiv 13 \pmod 7$

(iv) $-58 \equiv 707 \pmod 5$

(v) $58 \equiv 707 \pmod 5$

i. True: $3|(3+3) = 3|6$

ii. True: $9|(9000-0) = 9|9000$

iii. False: $7\nmid(13-44) = 7\nmid -31$

Problem 1

(b) Identify the statements that are true for mod using the equivalence definition!

(i) $-3 \equiv 3 \pmod{3}$

(ii) $0 \equiv 9000 \pmod{9}$

(iii) $44 \equiv 13 \pmod{7}$

(iv) $-58 \equiv 707 \pmod{5}$

(v) $58 \equiv 707 \pmod{5}$

i. True: $3|(3+3) = 3|6$
ii. True: $9|(9000-0) = 9|9000$
iii. False: $7\nmid(13-44) = 7\nmid-31$
iv. True: $5|(707+58) = 5|765$

Problem 1

(b) Identify the statements that are true for mod using the equivalence definition!

(i) $-3 \equiv 3 \pmod{3}$

(ii) $0 \equiv 9000 \pmod{9}$

(iii) $44 \equiv 13 \pmod{7}$

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(v) $58 \equiv 707 \pmod{5}$

i. True: $3|(3+3) = 3|6$

ii. True: $9|(9000-0) = 9|9000$

iii. False: $7\nmid(13-44) = 7\nmid -31$

iv. True: $5|(707+58) = 5|765$

v. False: $5|(707-58) = 5\nmid 649$

Proving Divisibility

"Unwrapping"

a ≡ b (mod n) ⟺ n|(b-a) ⟺ (b-a) = n * k

Divides

For integers x, y we say $x|y$ ("x divides y") iff there is an integer z such that $xz = y$.

Equivalence in modular arithmetic

Let $a \in \mathbb{Z}, b \in \mathbb{Z}, n \in \mathbb{Z}$ and $n > 0$.
We say $a \equiv b \ (mod\ n)$ if and only if $n|(b - a)$

Problem 3

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

Problem 3

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(1) Understand what this claim means
(2) Write your start and end goal
(3) Write the skeleton
(4) Fill in the skeleton

Problem 3

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(1) Understand what this claim means

$3 \mid 3$ and $3 \mid 3$ so $3 = 3$

Or

$3 \mid -3$ and $-3 \mid 3$ so $3 = -3$

(2) Write your start and end goal

(3) Write the skeleton

(4) Fill in the skeleton

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(1) Understand what this claim means

3 | 3 and 3 | 3 so 3 = 3

Or

3 | -3 and -3 | 3 so 3 = -3

(2) Write your start and end goal

Start: some a and b where a | b and b | a

End: show that a = b or a = −b

(3) Write the skeleton

(4) Fill in the skeleton

Problem 3

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

   (3) Write the skeleton

Problem 3

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(3) Write the skeleton

Suppose that for some arbitrary integers a and b where $a|b$ and $b|a$

…

…

…

So we get $b = -a$ or $b = a$

Since a and b were arbitrary, the claim holds

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a

By the definition of divides, we have b = ka and **a** = jb, for some integers k, j

…

…

…

So we get b = -a or b = a

Since a and b were arbitrary, the claim holds

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a

By the <u>definition of divides</u>, we have b = ka and **a** = jb, for some integers k, j

…

…

…

So we get b = -a or b = a

Since a and b were arbitrary, the claim holds

Can we prove something about k and j to get to b = -a or b = a ?

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a

By the definition of divides, we have b = ka and **a** = jb, for some integers k, j

Substituting b, a= j(ka)

…

…

So we get b = -a or b = a

Since a and b were arbitrary, the claim holds

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a

By the <u>definition of divides</u>, we have b = ka and **a** = jb, for some integers k, j

Substituting b, a= j(ka)

Dividing both sides by a, we get 1 = jk.

…

So we get b = -a or b = a

Since a and b were arbitrary, the claim holds

What do we need to say about k and j to get to b = -a or b = a ?

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a

By the <u>definition of divides</u>, we have b = ka and **a** = jb, for some integers k, j

Substituting b, a= j(ka)

Dividing both sides by a, we get 1 = jk.

We can say that 1/j = k

This expression only holds when j and k are <u>either -1 or 1</u>

1/3 ≠ Integer
1/1 = Integer

So we get b = -a or b = a

Since a and b were arbitrary, the claim holds

Problem 3

(a) Prove that if a | b and b | a, where a and b are integers, then a = b or a = −b.

(4) Fill in the skeleton

Suppose that for some arbitrary integers a and b where a|b and b|a
By the definition of divides, we have b = ka and **a** = jb, for some integers k, j
Substituting b, a= j(ka)
Dividing both sides by a, we get 1 = jk.

We can say that 1/j = k
k must be an integer and we must get an integer from 1/j
We know that j and k must be either 1 or -1
So we get b = -a or b = a
Since a and b were arbitrary, the claim holds

Proof By Cases

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(1) Understand what this claim means
(2) Write your start and end goal
(3) Write the skeleton
(4) Fill in the skeleton

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(1) Understand what this claim means

$(3)^2 \equiv 1 \pmod 4$

$(2)^2 \equiv 0 \pmod 4$

If you square an ***even*** *integer, you get* ***0*** *(mod 4)*

If you square an ***odd*** *integer, you get* ***1*** *(mod 4)*

(2) Write your start and end goal

(3) Write the skeleton

(4) Fill in the skeleton

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(1) Understand what this claim means

$(3)^2 \equiv 1 \pmod 4$

$(2)^2 \equiv 0 \pmod 4$

If you square an ***even*** *integer, you get* ***0*** *(mod 4)*

If you square an ***odd*** *integer, you get* ***1*** *(mod 4)*

(2) Write your start and end goal

Start: Some integer

End: Prove the integer^2 will be either ***0*** *(mod 4) or* ***1*** *(mod 4)*

(3) Write the skeleton

(4) Fill in the skeleton

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(3) Write the skeleton

Let n be an arbitrary integer

Case 1: n is even … $n^2 \equiv 0 \pmod 4$

Case 2: n is odd … $n^2 \equiv 1 \pmod 4$

…

…

In all cases $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

Since n was arbitrary, the claim holds

(4) Fill in the skeleton

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 1: n is even

Then $n = 2k$ for some integer k

…

…

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 1: n is even

Then n = 2k for some integer k

…

By the definition of divides so $4 | n^2$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

Work one step backwards to "unwrap"

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod{4}$ or $n^2 \equiv 1 \pmod{4}$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 1: n is even

Then $n = 2k$ for some integer k

Then $n^2 = (2k)^2 = 4k^2$

Since k is an integer, k^2 is an integer.

By the definition of divides, $4 \mid 4k^2$ so $4 \mid n^2$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod{4}}$

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 2: n is odd

Then n = 2k+1 for some integer k

…

…

…

…

n^2 **≡ 0 (mod 4)**

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 2: n is odd

Then n = 2k+1 for some integer k

…

…

…

…

By the definition of divides, **4|** n^2-1

Then by the definition of congruence, $n^2 \equiv 0 \pmod 4$

Work one step backwards to "unwrap"

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 2: n is odd

Then n = 2k+1 for some integer k

…

…

…

So we can say that 4 * j = $\mathbf{n^2 - 1}$

By the definition of divides, **4|** $\mathbf{n^2-1}$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

Work one step backwards to "unwrap"

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 2: n is odd

Then n = 2k+1 for some integer k

Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1$

…

…

So we can say that 4 * j = $\mathbf{n^2 - 1}$

By the definition of divides, **4|** $\mathbf{n^2-1}$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 2: n is odd

Then $n = 2k+1$ for some integer k

Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1$

So $\mathbf{n^2 - 1} = 4(k^2 + k)$

Since k is an integer, we can say $j = k^2 + k$ where j is an integer.

So we can say that $4 * j = \mathbf{n^2 - 1}$

By the definition of divides, $\mathbf{4 | n^2-1}$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

Problem 5: Fair and Square

(a) Prove that for all integers n, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$

(4) Fill in the skeleton

Let n be an arbitrary integer

Case 1: n is even

Then n = 2k for some integer k

Then $n^2 = (2k)^2 = 4k^2$

Since k is an integer, k^2 is an integer.

By the definition of divides, $4 \mid 4k^2$ so $4 \mid n^2$

Then by the definition of congruence, $n^2 \equiv 0 \pmod 4$

Case 2: n is odd

Then n = 2k+1 for some integer k

Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1$

So $\mathbf{n^2 - 1} = 4(k^2 + k)$

Since k is an integer, we can say $j = k^2 + k$ where j is an integer.

So we can say that $4 * j = \mathbf{n^2 - 1}$

By the definition of divides, $\mathbf{4 \mid n^2-1}$

Then by the definition of congruence, $\mathbf{n^2 \equiv 0 \pmod 4}$

In either case, $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$. Since n was arbitrary, the claim holds

Proofs by Contrapositive

Some claims are hard to prove directly!

- Sometimes you will run into claims that, because of the way they are structured, will be time consuming or difficult to prove.
- Recall in lecture, you attempted to prove that if the square of an integer is even, the integer must also be even.
- It was problematic to prove this because you had to deal with square roots.

Luckily, we can manipulate implications to put them into a form that is easier to solve!!

Why does Proof by Contrapositive work?

Consider the following truth table:

P	Q	$\neg P$	$\neg Q$	$P \to Q$	$\neg Q \to \neg P$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Note that, when the assignments of P and Q are such that $P \to Q$ is true, the assignments of their negation's *must also be such* that $\neg Q \to \neg P$ is also true. They are logically equivalent statements! (You can also do a four-step chain of equivalence to show this.)

Problem 6

For any integer j, if 3j+1 is even, then j is odd

(a) Write the predicate logic of this claim

Odd(x) := x is Odd

Even(x) := x is Even

(b) Write the contrapositive of this claim

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$\forall j\ (\text{Even}(3j+1) \rightarrow \text{Odd}(j))$

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$\forall j \, (Even(3j+1) \rightarrow Odd(j))$

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For any integer j, if j is even, 3k+1 is odd
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(c) Determine which claim is easier to prove, then prove it!

Take around 5 minutes and write an English proof for part (a) or (b)

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We will prove the contrapositive of this claim.

Let j be an arbitrary even integer.
By the definition of even $j=2k$ for some integer k.
Then by Algebra, $3j + 1 = 3(2k) + 1 = 2(3k) + 1$
Since k is an integer, under closure of multiplication, 3k is an integer.
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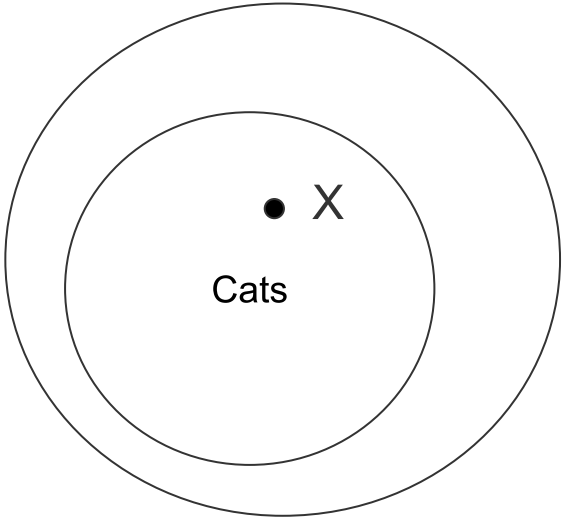
Let j be an arbitrary even integer.
By the definition of even j=2k for some integer k.
Then by Algebra, $3j + 1 = 3(2k) + 1 = 2(3k) + 1$
Since k is an integer, under closure of multiplication, 3k is an integer.
Therefore $2(3k) + 1$ takes the form of an odd integer so $3j + 1$ must be odd.
Since j was arbitrary and we have shown the contrapositive, the claim holds!

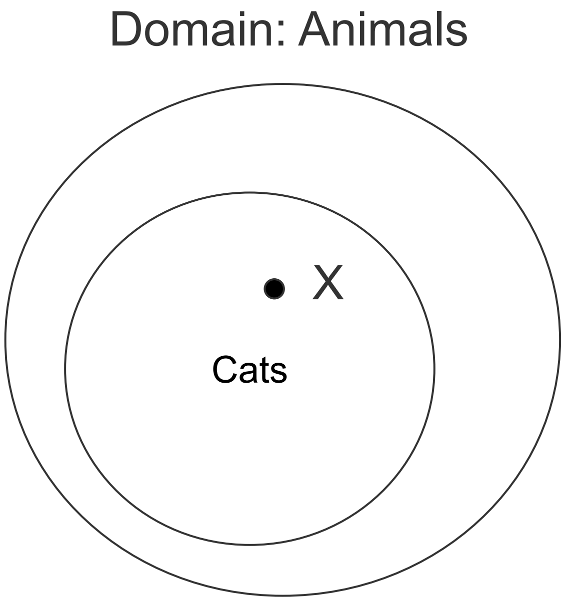
Side Note: What exactly *is* arbitrary?

Domain: Animals



- Arbitrary is a word we use to describe an unspecified member of our domain. You can think of it as simultaneously being *any* and *all* members.
- When we attempt to prove universal claims, we must prove them with arbitrary variables. This is how we can know a claim holds for all members of the domain.

Side Note: What exactly *is* arbitrary?



- Consider the following claim: $\forall x[Cat(x) \rightarrow CuteInHolidaySweater(x)]$
- We would go about proving this claim by first defining x to be an *arbitrary cat.*
- If we can show that x does in fact look cute in a holiday sweater, this would mean that we have shown that *all members of the domain Cats* must also look cute in a holiday sweater.
- In other words, we have proven a relationship between the *property* of being a cat and the *property* of looking cute in a holiday sweater.

Side Note: What exactly *is* arbitrary?

An arbitrary cat, X

- X is an arbitrary cat, and X looks cute in a holiday sweater. So we know that for any element of our domain X, if it is a cat, then it must also look cute in a holiday sweater.

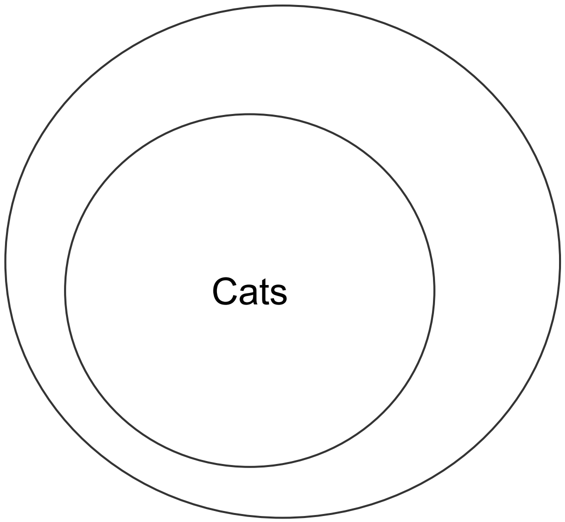
Side Note: What exactly *is* arbitrary?

More arbitrary cats

- We can pull as many elements from our domain as we like, in order for a universal claim to be true, it must be true for all of them.
- Claim proven!
- But what about existential claims?

Side Note: What exactly *is* arbitrary?

Domain: Animals



- Consider the following claim: $\exists x[Cat(x) \wedge DrinksBoba(x)]$
- Note that this is an existential claim. All we have to do is show that *at least one* example member of our domain fulfills these criteria in order for the claim to hold.
- Proving existentials is easier than proving universals, you simply furnish an example!

Side Note: What exactly *is* arbitrary?

A specific animal, Bagel

- When furnishing examples for an existential claim, we usually say “Consider… blank.”
- So, consider the following member of our domain, Bagel.
- Bagel is a cat, and Bagel drinks boba.
- This is enough information for us to know that the existential claim $\exists x[Cat(x) \wedge DrinksBoba(x)]$ holds. It *does not* have to extend to all other members of the domain, though it can.

That's All Folks

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