Section 09: Models of Computation

1. CFGs

Write a context-free grammar to match each of these languages.

(a) All binary strings that start with 11.

(b) All binary strings that contain at most one 1.

(c) All strings over 0, 1, 2 with the same number of 1s and 0s and exactly one 2.
Hint: Try modifying the grammar from Section 8 2c for binary strings with the same number of 1s and 0s (You may need to introduce new variables in the process).

2. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

3. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

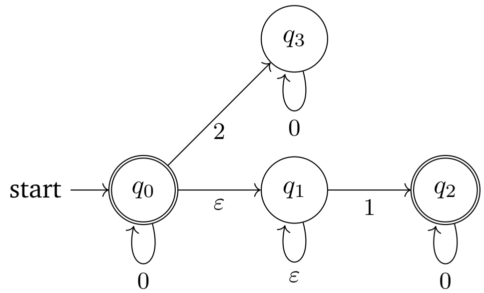
(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

4. NFAs

(a) What language does the following NFA accept?



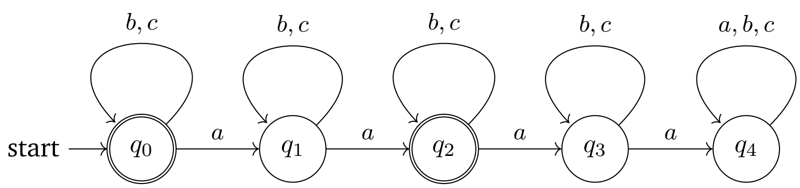
(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

5. DFAs & Minimization

Note: We will not test you on minimization, although you may optionally read the extra slides and do this problem for fun

(a) Convert the NFA from 1a to a DFA, then minimize it.

(b) Minimize the following DFA:



6. Onto & One-to-One

Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is onto but not one-to-one. Be specific.

7. Proving Onto

(a) Suppose that $f : \mathbb{Z}^2 \to \mathbb{Z}$ is defined by $f(x, y) = xy + yx^2 - x^2$. Prove that f is onto, where $\mathbb{Z}^2 = \mathbb{Z}$ x $\mathbb{Z}$ which represents all possible ordered pairs of integers.

(b) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that f is onto.