CSE 312: Foundations of Computing II
Quiz Section #5

1. Let the random variable X be the sum of two independent rolls of a fair die. Calculate $\mathrm{E}[X]$ using linearity of expectation. Compare your answer and the ease of computing it to the corresponding problem in last week's quiz section worksheet.

2. There are 20 Statistics students and 20 Mathematics students. They are randomly split into 20 study pairs, with 2 students per study pair. All such pairings are equally probable. Let the random variable X denote the number of pairs consisting of 1 Statistics student and 1 Mathematics student. Find $\mathrm{E}[X]$. (Notice that the pairs formed are not independent events: if A is paired with B, then C cannot be paired with B.)

3. You have 10 pairs of socks (so 20 socks in total), with each pair being a different color. You put them in the washing machine, but the washing machine eats 4 of the socks chosen at random. Every subset of 4 socks is equally probable to be the subset that gets eaten. What is the expected number of complete pairs of socks that you have left?

4. Find the expected number of bins that remain empty when m balls are distributed into n bins randomly and independently. For each ball, each bin has an equal probability of being chosen. (Notice that two bins being empty are not independent events: if one bin is empty, that decreases the probability that the second bin will also be empty. This is particularly obvious when $n = 2$ and $m > 0$.)

5. At a reception, n people give their hats to a hat-check person. When they leave, the hat-check person gives each of them a hat chosen at random from the hats that remain. What is the expected number of people who get their own hats back? (This is closely related to, but much simpler than, the challenge problem from the worksheet from quiz section #3. Notice that the hats returned to two people are not independent events: if a certain hat is returned to one person, it cannot also be returned to the other person.)