CSE 312: Foundations of Computing II
Quiz Section #7: Exponential distribution

Recall the probability density function for $X \sim \text{Exp}(\lambda)$:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{, if } x \geq 0 \\ 0 & \text{, if } x < 0 \end{cases}.$$

1. Starting from the probability density function, prove that $\text{E}[X] = 1/\lambda$. (Hint: use integration by parts.)
2. Starting from the probability density function, prove that $\text{P}(X \geq t) = e^{-\lambda t}$, for $t \geq 0$. As a corollary, show that the cumulative distribution function for X is $F(t) = 1 - e^{-\lambda t}$.
3. Prove the memorylessness property for exponential distributions: If s and t are nonnegative, then $\text{P}(X \geq s + t \mid X \geq s) = \text{P}(X \geq t)$.