

Review of Important Distributions

1. Discrete
2. Continuous

Discrete Random Variables

Discrete Uniform Distribution

Definition: A random variable that takes any integer value in an interval with equal likelihood

Example: Choose an integer uniformly between 0 and 10

Parameters: integers a , b (lower and upper bound of interval)

Properties:

$$E[X] = \frac{a + b}{2}$$

$$\text{Var}(X) = \frac{(b - a)(b - a + 2)}{12}$$

$$\text{pdf: } P(X=k) = \frac{1}{b-a+1} \text{ if } k \in [a, b], 0 \text{ otherwise}$$

Bernoulli Distribution

Definition: value 1 with probability p , 0 with probability $1-p$

Example: coin toss ($p = 1/2$ for fair coin)

Parameters: p

Properties:

$$E[X] = p$$

$$\text{Var}(X) = p(1-p)$$

Binomial Distribution

Definition: sum of n independent Bernoulli trials, each with parameter p

Example: number of heads in 10 independent coin tosses

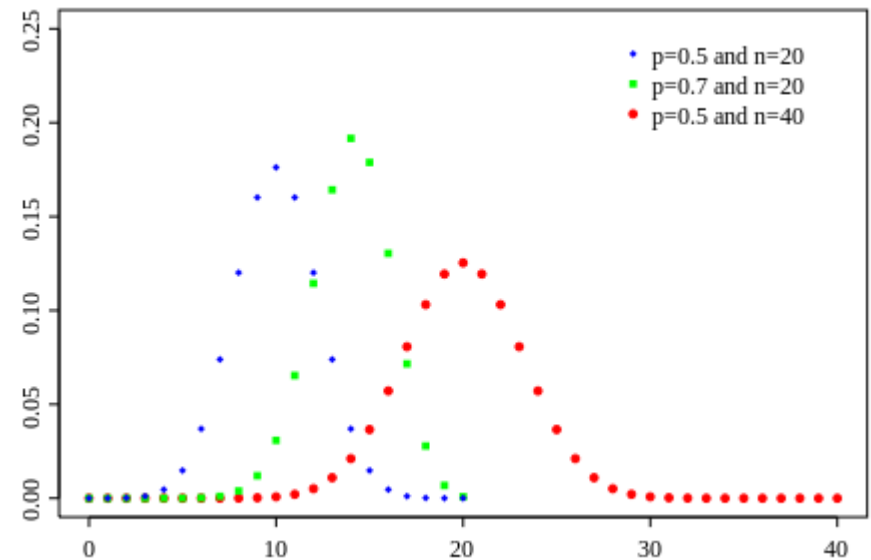
Parameters: n, p

Properties:

$$E[X] = np$$

$$\text{Var}(X) = np(1-p)$$

$$\text{pmf: } P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$



Poisson Distribution

Definition: number of events that occur in a unit of time, if those events occur independently at an average rate λ per unit time

Example: # of cars at traffic light in 1 minute, # of deaths in 1 year by horse kick in Prussian cavalry

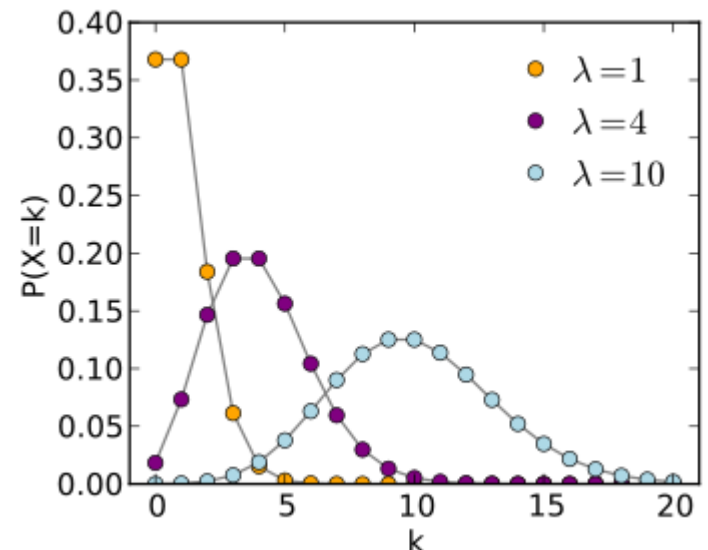
Parameters: λ

Properties:

$$E[X] = \lambda$$

$$\text{Var}(X) = \lambda$$

$$\text{pmf: } P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$



Geometric Distribution

Definition: number of independent Bernoulli trials with parameter p until and including first success (so X can take values 1, 2, 3, ...)

Example: # of coins flipped until first head

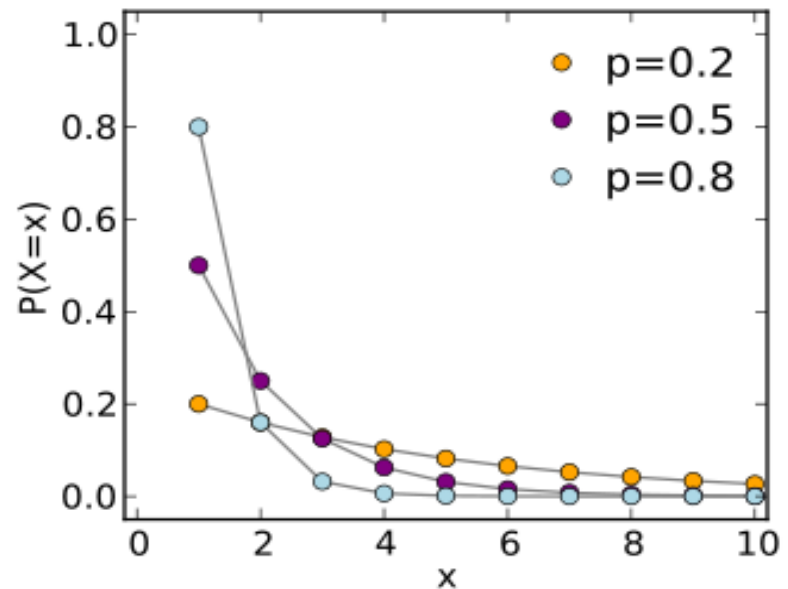
Parameters: p

Properties:

$$E[X] = \frac{1}{p}$$

$$\text{Var}(X) = \frac{1-p}{p^2}$$

pmf: $P(X = k) = (1 - p)^{k-1}p$



Hypergeometric Distribution

Definition: number of successes in n draws (without replacement) from N items that contain K successes in total

Example: An urn has 10 red balls and 10 blue balls. What is the probability of drawing 2 red balls in 4 draws?

Parameters: n, N, K

Properties:

$$E[X] = n \cdot \frac{K}{N}$$

$$\text{Var}(X) = n \cdot \frac{K(N-K)(N-n)}{N^2(N-1)}$$

pmf:
$$P(X = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}$$

Think about the pmf; we've been doing it for weeks now: ways-to-choose-successes times ways-to-choose-failures divided by ways-to-choose-all.

Also, consider that the binomial dist. is the with-replacement analog of this.

Continuous Random Variables

Continuous Uniform Distribution

Definition: A random variable that takes any real value in an interval with equal likelihood

Example: Choose a real number (with infinite precision) between 0 and 10

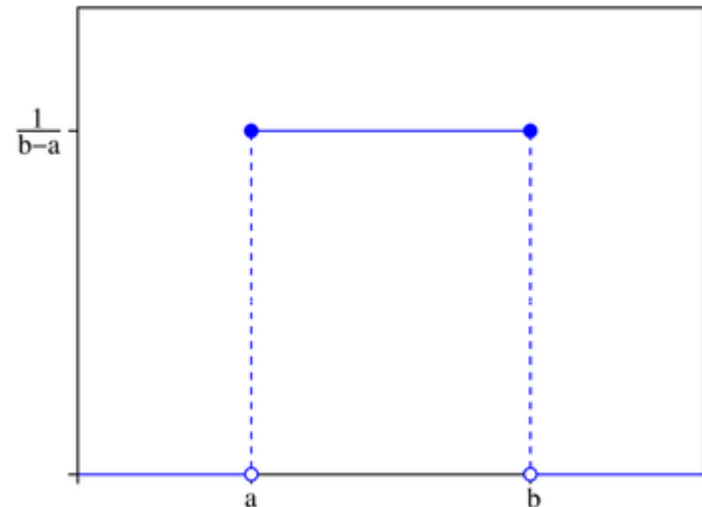
Parameters: a , b (lower and upper bound of interval)

Properties:

$$E[X] = \frac{a + b}{2}$$

$$\text{Var}(X) = \frac{(b - a)^2}{12}$$

pdf: $f(x) = \frac{1}{b-a}$ if $x \in [a, b]$, 0 otherwise



Exponential Distribution

Definition: Time until next event in Poisson process

Example: How long until the next soldier is killed by horse kick?

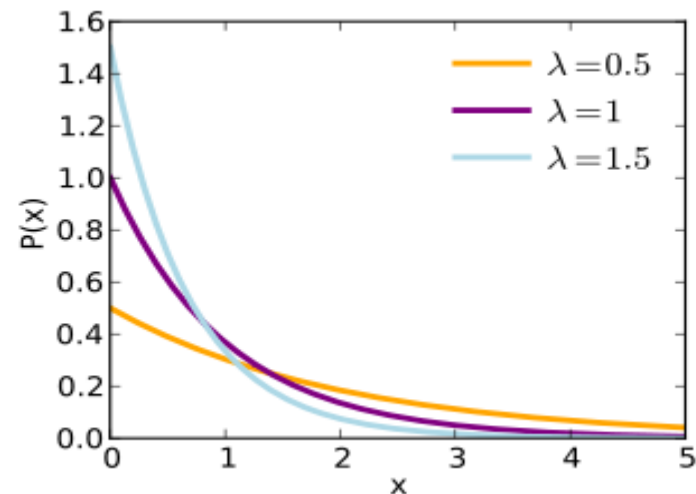
Parameters: λ , the average number of events per unit time

Properties:

$$E[X] = \frac{1}{\lambda}$$

$$\text{Var}(X) = \frac{1}{\lambda^2}$$

pdf: $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, 0 for $x < 0$



Normal Distribution

Description: Classic bell curve

Example: Quantum harmonic oscillator ground state (exact)
Human heights, binomial random variables (approximate)

Properties: μ , σ^2

$$E[X] = \mu$$

$$\text{Var}(X) = \sigma^2$$

$$\text{pdf: } f(x) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

