CSE 312: Foundations of Computing II
Quiz Section #5: Midterm review

1. Let A and B be events in the same sample space that each have nonzero probability. For each of the following statements, state whether it is always true, always false, or it depends on information not given.

   (a) If A and B are mutually exclusive, then they are independent.

   (b) If A and B are independent, then they are mutually exclusive.

   (c) If $P(A) = P(B) = 0.75$, then A and B are mutually exclusive.

   (d) If $P(A) = P(B) = 0.75$, then A and B are independent.

2. Let X be a random variable having $E[X] = \mu$ and $\text{Var}(X) = \sigma^2$. Find the expectation and variance of a new random variable Y, where $Y = (X - \mu)/\sigma$. (Y is called the "standardized" version of X.)

3. Given any set of 18 integers, show that one may always choose two of them so that their difference is divisible by 17.

4. (This problem is identical to a problem from HW1, except that "=" has been replaced by "≤".)

   Consider the following inequality: $a_1 + a_2 + a_3 + a_4 + a_5 \leq 100$. A solution to this inequality over the nonnegative integers is a choice of a nonnegative integer for each of the 5 variables a_1, a_2, a_3, a_4, a_5 that satisfies the inequality. To be different, two solutions have to differ on the value assigned to some a_i. How many different solutions are there to the inequality?

5. You roll three fair dice, each with a different numbers of faces: die 1 has six faces (numbered $1 \ldots 6$), die 2 has eight faces (numbered $1 \ldots 8$), and die 3 has twelve faces (numbered $1 \ldots 12$). Let the random variable X be the sum of the three values rolled. What is $E[X]$?

6. How many integers in $\{1, 2, \ldots, 360\}$ are divisible by one or more of the numbers 2, 3, and 5?

7. Recall that a Schnapsen deck has 4 suits with 5 cards in each suit. Suppose a deck of Schnapsen cards is shuffled well and then dealt into 5 piles of 4 cards each. Let E_i refer to the event that pile i has exactly one spade. Compute the probability $P(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5)$.

8. You are trying to diagnose the probability that a patient with a positive blood sugar test result has diabetes, even though she is in a low risk group. The probability of a woman in this group having diabetes is 0.8%. 90% of women with diabetes will test positive in the blood sugar test. 7% of women without diabetes will test positive in the blood sugar test. Your patient tests positive in the blood sugar test. What is the probability that she has diabetes?

9. n people at a reception give their hats to a hat-check person. When they leave, the hat-check person gives each of them a hat chosen at random. What is the expected number of people who get their own hat back? (This is closely related to, but much simpler than, the challenge problem from the worksheet from quiz section #2.)

10. A very long multiple choice exam has 4 choices for each question. Charlie has studied enough so that he knows the correct answer for 1/2 of the questions, for an additional 1/4 of the questions he

can eliminate one choice and chooses randomly and uniformly among the other three, and for the remaining 1/4 of the questions he chooses randomly and uniformly among all four answers.

As the teacher, you want to determine how many answers the student actually knows. For a randomly chosen question, if Charlie answers it correctly, what is the probability he knew the answer?