

My office hour today is cancelled.

Let  $X$  be the number of heads when a coin with  $P(\text{heads}) = p$  is flipped  $n$  times independently. Find  $E[X]$  and  $\text{Var}(X)$ .

For  $1 \leq i \leq n$ , let  $X_i = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ flip is heads} \\ 0, & \text{if } i^{\text{th}} \text{ flip is tails} \end{cases}$  "indicator" r.v.  
$$X = \sum_{i=1}^n X_i$$

$$E[X_i] = 1 \cdot P(X_i=1) + 0 \cdot P(X_i=0) \\ = P(i^{\text{th}} \text{ flip is heads}) = p$$

$$E[X_i^2] = 1^2 \cdot P(X_i=1) + 0^2 \cdot P(X_i=0) = p$$

$$\text{Var}(X_i) = E[X_i^2] - (E[X_i])^2 = p - p^2 = p(1-p)$$

$$E[X] = E\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n E[X_i] = np$$

$$\text{Var}(X) = \text{Var}\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \text{Var}(X_i) \quad (\text{independence!}) \\ = np(1-p).$$