MLE Steps
1. Find likelihood and log-likelihood.
2. Differentiate and set to 0, solve.
3. Verify it is a maximum by showing the second derivative is negative (and checking endpoints).

1. (MLE) Suppose $x_1, \ldots, x_n$ are iid samples from a distribution with density

$$f_X(x;\theta) = \begin{cases} \dfrac{\theta x^{\theta-1}}{2^\theta}, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$

Find the MLE for θ.

2. (Bias) Suppose $X_1, \ldots, X_n$ are iid samples from a continuous uniform distribution, $Unif(0,\theta)$. Consider the estimator $\hat{\theta} = \frac{3}{n}\sum_{i=1}^{n} X_i$. Is $\hat{\theta}$ unbiased? If not, find a scalar c such that $c\hat{\theta}$ is an unbiased estimator.

3. (Continuous Distributions). Let X have the following density:

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

a) Find $E\left[\frac{1}{X}\right]$.

b) What is $P(X = 0.5)$?

4. (Counting) Suppose we have a standard 52-card deck and are dealt 5 cards. What is the probability we draw a full house? (3 of a kind, and 2 of a kind) (Ex. AAA22, J3J33, etc)