CSE 312: Foundations of Computing II
Quiz Section #8: Central Limit Theorem

1. Before putting any bets down on roulette, you watch 100 rounds, each of which results in an integer between 1 and 36. You count how many rounds have a result that is odd and, if the count exceeds 55, you decide the roulette wheel is unfair. Assuming the roulette wheel is fair, approximate the probability that you make the wrong decision.

2. A factory produces X_i gadgets on day i, where the X_i are independent and identically distributed random variables, each with mean 5 and variance 9.

    (a) Approximate the probability that the total number of gadgets produced in 100 days is less than 440.

    (b) Approximate the greatest value of n such that $P(X_1 + X_2 + \cdots + X_n \geq 5n + 200) \leq 0.05$.

3. (a) A fair coin is tossed 50 times. Use the Central Limit Theorem to estimate the probability that fewer than 20 of those tosses come up heads.

    (b) A fair coin is tossed until it comes up heads for the 20th time. Use the Central Limit Theorem to estimate the probability that more than 50 tosses are needed. (Hint: you will need the mean and variance of a geometric random variable, which you can find in Example 2.15 of the text.)

    (c) Compare your answers from parts (a) and (b). Why are they close but not exactly equal?

4. Suppose 59 percent of voters favor Proposition 666. Use the Normal approximation to estimate the probability that a random sample of 100 voters will contain:

    (a) at most 50 in favor.

    (b) between 54 and 64 (inclusive) in favor.

    (c) fewer than 72 in favor.

5. Each day, the probability your computer crashes is 10%, independent of every other day. Approximate the probability of at least 87 crash-free days out of the next 100 days.

6. Let $X \sim \text{Exp}(\lambda)$ and $k > 1/\lambda$.

    (a) Use Markov's inequality to bound $P(X \geq k)$.

    (b) Use Chebyshev's inequality to bound $P(X \geq k)$.

    (c) What is the exact formula for $P(X \geq k)$?

    (d) For $\lambda k \geq 3$, how do the bounds given in parts (a), (b), and (c) compare?