Basic Notions of Probabilistic Reasoning

p(A)

is the **probability** that proposition A is true.

$p(L = \langle x,y,\theta \rangle) = 0.1$

the **probability** of the robot **being at location** $\langle x,y,\theta \rangle$ **given no other information** is 0.1

L

is denoted as a **random variable**

Probability Distributions

- **Random variables** can **have multiple values**:
  - boolean: *True, False*
  - multi-valued: *high*, *low*, *medium*
  - continuous

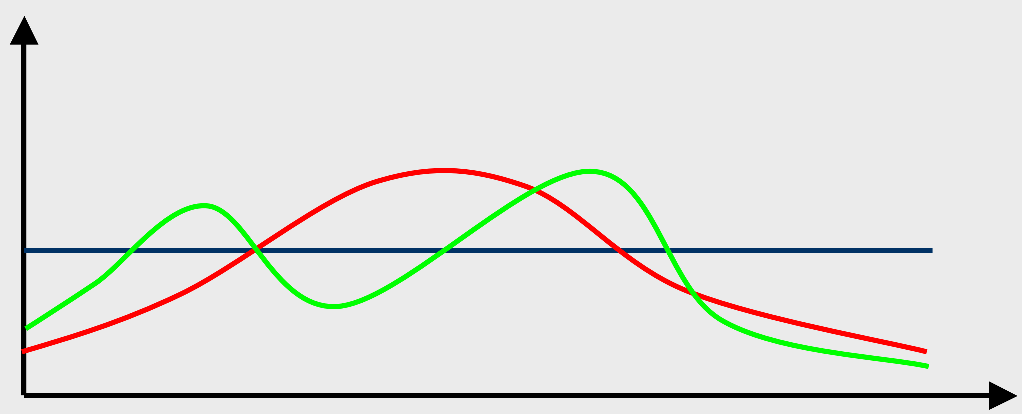
- A **probability distribution** specifies the **probabilities for all possible values** of **a random variable**.

Discrete vs. Continuous

- Discrete:

$$P(Room) = \langle 0.7, 0.2, 0.08, 0.02 \rangle$$

- Continuous:



Conditional Probability

- The term $p(A)$ is a **prior probability**
- Suppose we also know B
- We use

$$p(A|B)$$

to describe the **probability of *A* given** all we know is ***B***.

The Product Rule

Product rule:

$$p(A \wedge B) = p(A \mid B)\, p(B)$$

Conditional probabilities can be expressed in terms of unconditional probabilities!

Bayes Formula

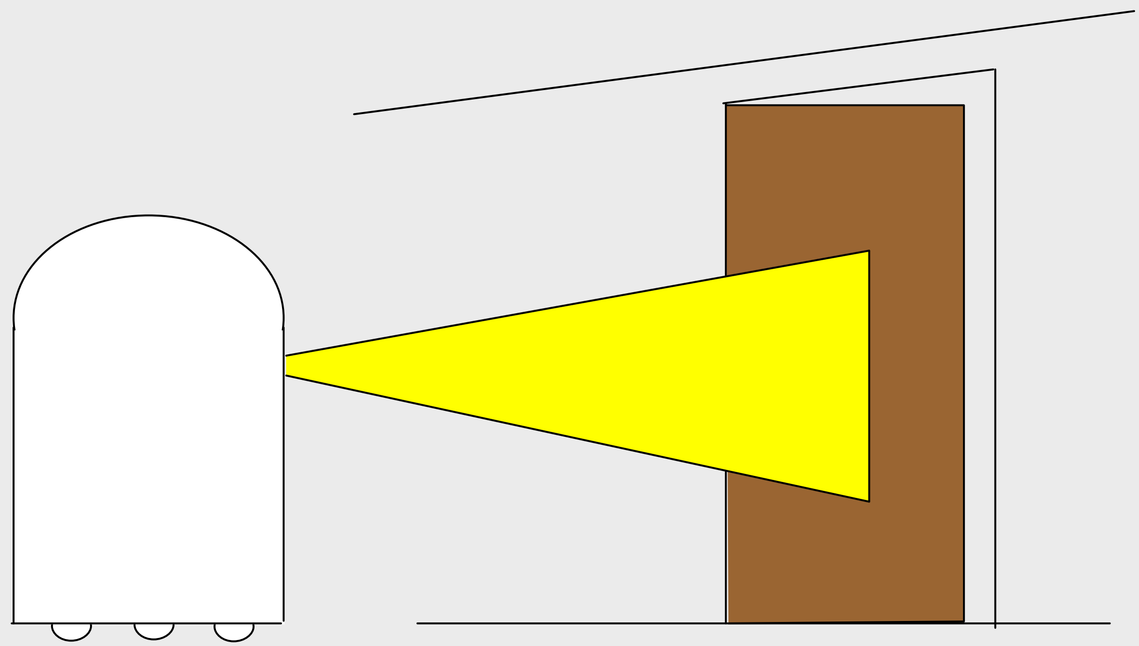
$$p(A \wedge B) = p(A|B)\,p(B) = p(B|A)\,p(A)$$

$$\Rightarrow$$

$$p(A|B) = \frac{p(B|A)\;p(A)}{p(B)}$$

A Simple Example: State Estimation

- Suppose a robot obtaines measurement *s*
- What is *p(doorOpen|s)?*



Causal vs. Diagnostic Reasoning

- *$p(open|s)$ is diagnostic*
- *Often causal knowledge like*

$$p(s \mid open)$$

count frequencies!

is easier to obtain.

- *Application of Bayes rule:*

$$p(open \mid s) = \frac{p(s \mid open)\, p(open)}{p(s)}$$

Normalization

$$p(open \mid s) = \frac{p(s \mid open)\, p(open)}{p(s)}$$

$$p(\neg open \mid s) = \frac{p(s \mid \neg open)\, p(\neg open)}{p(s)}$$

$$p(open \mid s) + p(\neg open \mid s) = 1$$

$$\Rightarrow$$

$$p(s) = p(s \mid open)\, p(open) + p(s \mid \neg open)\, p(\neg open)$$

$$\Rightarrow$$

$$p(open \mid s) = \frac{p(s \mid open)\, p(open)}{p(s \mid open)\, p(open) + p(s \mid \neg open)\, p(\neg open)}$$

Example

- $p(s|open) = 0.6$ $\qquad p(s|\neg open) = 0.3$
- $p(open) = p(\neg open) = 0.5$

$$p(open \mid s) = \frac{p(s \mid open)\, p(open)}{p(s \mid open)\, p(open) + p(s \mid \neg open)\, p(\neg open)}$$

$$p(open \mid s) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

s raises the probability, that the door is open.

Integrating a second Measurement ...

- New measurement s_2
- $p(s_2|open) = 0.5$ $\qquad$ $p(s_2|\neg open) = 0.6$

$$p(open \mid s_2 s_1) = \frac{p(s_2 \mid open)\, p(open \mid s_1)}{p(s_2 \mid open)\, p(open \mid s_1) + p(s_2 \mid \neg open)\, p(\neg open \mid s_1)}$$

$$p(open \mid s_2 s_1) = \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

s_2 lowers the probability, that the door is open.