CSE 321: Discrete Structures
Assignment #2
Due: Wednesday, October 15

Reading Assignment: Rosen, 5th edition: Sections 1.3-1.8 pp. 233-236, 2.4-2.5. (If you have the 4th edition, these are 1.3-1.6, pp. 76-78, 3.1, 2.3-2.4.)

Problems:

1. Let $Q(x, y)$ be the statement "x has been a contestant on y". Express the following sentences in terms of $Q(x, y)$, quantifiers and logical connectives, where the universe of discourse for x is the set of all students at your school and the universe of discourse for y is the set of all quiz shows on television.
   - No student at your school has ever been a contestant on a television quiz show.
   - Every television quiz show has had a student from your school as a contestant.
2. Let $F(x, y)$ be the statement "x can fool y",where the universe of discourse is the set of all people in the world. Use quantifiers to express each of the following statements.
   - No one can fool both Fred and Jerry.
   - No can fool himself or herself.
   - There is someone who can fool exactly one person besides himself or herself.
3. Give the negation of each of the following statements. (If the statement is in English, give the negation in English.):
   - All good students study hard.
   - No males give birth to their young.
   - No students in mathematics are unable to use a computer.
   - $\forall x \exists y\ x = y^2$
4. Prove that the square of an even number is an even number using
   - a direct proof
   - a proof by contradiction
5. Prove that if n is an integer and $3n + 2$ is even, then n is even using an indirect proof.
6. Let $Q(A, B)$ be the statement $A \subseteq B$. If the universe of discourse for both A and B is all sets of integers, what are the truth values of the following? Justify your answers.
   - $(\forall A)(\exists B)Q(A, B)$
   - $(\forall B)(\exists A)Q(A, B)$

- $(\exists A)(\forall B)Q(A, B)$
- $(\forall A)(\forall B)Q(A, B)$

7. Which of the following statements are true?

   - $\{x\} \subseteq \{x\}$
   - $\{x\} \in \{x, \{x\}\}$
   - $\{x\} \in \{x\}$
   - $\{x, \{x\}\} \subseteq \mathcal{P}(\{x\})$

8. Carefully prove the following implications.

   - $(A \cup B = B) \to (A \subseteq B)$
   - $(A \cap B = A) \to (A \subseteq B)$