1. Prove that if n is an odd positive integer, then n^4 ?1 (mod 16).

2. A **perfect number** is a positive integer that equals the sum of its proper divisors (that is, devisors other than itself). Show that 6, 28, and 496 are perfect.

3. Prove that $1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ whenever n is a nonnegative integer.

4. Show that $2^n > n^2$ whenever n is a positive integer greater than 4.

5. An automatic teller machine has only \$20 bills and \$50 bills. Which amount of money can the machine dispense, assuming the machine has a limitless supply of there two denominations of bills? Prove your answer using a form of mathematical induction.

6. A multiple-choice test contains ten questions. There are four possible answers for each question.

    a. How many ways can a student answer the questions on the test if every question is answered?
    b. How many questions can a student answer the questions on the test if the student can leave answers blank?

1. $n^4 - 1 = (2k+1)^4 - 1 = 16k^4 + 32k^3 + 24k^2 + 8k = 8k(k+1)(2k^2 + 2k + 1)$ One of *k* or *k+1*is even, so 16 divides n^4*-1*.

2. 1+2+3=6
1+2+4+7+14=28
1+2+4+8+16+31+62+124+248=496

3. Let *P(n)* be "$1^2+3^2+\ldots+(2n+1)^2=(n+1)(2n+1)(2n+3)/3$."
Basis Step: P(0) is true since $1^2=1=(0+1)(2*0+1)(2*0+3)/3$
Inductive Step: Assume that *P(k)* is true. Then

$$1^2 + 3^2 + \ldots + (2k+1)^2 + (2(k+1)+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2$$

$$= (2k+3)\left[\frac{(k+1)(2k+1)}{3} + (2k+3)\right] = \frac{(2k+3)(2k^2+9k+10)}{3}$$

$$= \frac{(2k+3)(2k+5)(k+2)}{3} = \frac{((k+1)+1)(2(k+1)+1)(2(k+1)+3)}{3}$$

4. Let *P(n)* be "$2^n > n^2$."
Basis Step: *P(5)* is true since $2^5 = 32 > 25 = 5^2$.
Inductive Step: Assume that *P(k)* is true, that is, $2^k > k^2$. Then

$2^{k+1} = 2 * 2^k > k^2 + k^2 > k^2 + 4k \geq k^2 + 2k + 1 = (k+1)^2$ since *k*>*4*.

5. All multiples of $10 greater than or equal to $40 can be formed as wells as $20. Let *P (n)* be the statement that *10n* dollars can be formed. *P(4)* is true since $40 can be formed by using two $20s.Now assume that *P(k)* is true with *k ? 4*. If a $50 bill is used to form *10k* dollars, replace it by three $20 bills to obtain *10(k+1)* dollars. Otherwise, at least two $20 bills were used since *10k* is at least $40. Replace two $20 bills with a $50 bill to obtain *$10(k+1)*. This shows that *P(k+1)* is true.

6. a) 4^{10}
b)5^{10}