CSE 321: Discrete Structures Spring 2006

PROBLEM SET 3

Due Friday, April 21, in class

Reading: Sections 2.4 – 2.6, 3.3.

1. Prove that $\sum_{0 \leq i \leq k} 2^i = 2^{k+1} - 1$. Hint: Define $S = \sum_{0 \leq i \leq k} 2^i$, and consider the quantity $2S$.

2. Section 2.4, Exercise 40.

3. Using only your brain, pencil and paper (e.g., no calculator) to find the rightmost digit (digit in unit place) when the following numbers are written in decimal. Show all your intermediate steps as proof that you used your brain instead of a calculator:

   (a) 32^{631}

   (b) $7^{7^{14}}$

   (c) $1! + 2! + 3! + \cdots + 100!$

   (If you follow the method I outlined in class, you should never have to compute a product greater than $9 \cdot 9$.)

4. Use Euclid's algorithm (and the extended Euclidean algorithm) to compute the following showing all the intermediate steps.

   (a) $\gcd(2274, 174)$

   (b) The inverse of $144 \mod 233$.

5. (a) Let a, b be positive integers. Define $S_{a,b}$ to be the set of all **positive** integers that can be written in the form $sa + tb$ for integers s, t. In class, using the Extended Euclidean algorithm, we showed that $\gcd(a, b) \in S_{a,b}$. Prove that the smallest element in $S_{a,b}$ is in fact equal to $\gcd(a, b)$.

   (b) Prove that the linear equation $ax + by = c$ where a, b, c are integers and $a \neq 0$ and $b \neq 0$ has a solution in integers (x, y) if and only if $\gcd(a, b) | c$.
   (Do not forget to show both directions of the "if and only if")