CSE 321: Discrete Structures   Spring 2006

Reading: Chapter 5.

1. Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where k is an integer.

2. What is the probability that a five-card poker hand contains exactly 2 hearts and 2 spades?

3. Section 5.1, Exercise 40

4. Section 5.2, Exercise 24

5. Section 5.2, Exercise 26

6. Section 5.2, Exercise 28

7. Suppose we choose randomly and independently two subsets A and B from the set of all possible non-empty subsets of $\{1, 2, \ldots, n\}$. What is the probability that $\min(A) = \min(B)$ (where $\min(A)$ denotes the minimum number from the set A).

8. Great Expectations:

   (a) Suppose a 6-sided fair dice is rolled. Let the random variable X be the value showing. What is the expectation of X? Suppose two fair 6-sided dice are rolled independently. Let Y be the random variable which is the sum of the two values showing. What is the expectation of Y? Let Z be the random variable which is the minimum of the two values showing. What is the expected value of Z?

   (b) Suppose that a fair coin is tossed 1000 times. Let X be the random variable which is the number of flips i in which the coin takes the same value in both flip i and $i + 1$. What is the expected value of X? (Just to clarify, in the sequence $HHHH$, X is 3, and in the sequence $THHHTT$, X is also 3, etc.)

9. Itty-Bitty Airlines accepted 12 reservations for a small commuter plane with 10 seats. They know that 6 reservations went to regular commuters who will show up for sure. Each of the other 6 passengers will show up with a 50% chance, independently of each other. Show your work.

   (a) Find the probability that the flight will be overbooked.

   (b) Itty-Bitty never ever cancels flights – unless too few people show up. Suppose Itty-Bitty's management has decreed that the flight will be cancelled unless there are at least 8 passengers on board. What is the probability that the flight will get cancelled?

   (c) Find the average number of passengers turned away due to cancellations.

   (d) Find the average number of passengers turned away due to overbooking.

   (e) Suppose Itty-Bitty has to pay $ 100 to each person who cannot fly because his flight is overbooked, and has to pay $75 to each person (who shows up) whose flight gets cancelled. How much should Itty-Bitty charge per ticket to expect to break even?

10. (**Extra Credit**) Suppose you play the following game. Your rival has 100 blank cards, and on each of them she writes an arbitrary integer at will (and never repeats an integer on two cards). Then the cards are perfectly shuffled, and the shuffled deck is laid face down on a table so that the numbers are not visible. You start removing cards from the top one by one and look at their numbers. After turning any of the cards, you can end the game. You win the game if and only if the last card you turned has the largest number among all the cards (those already turned but also those still lying on the table).

The rival proposes the following stakes to you: if you win she will give you 20 dollars, and if you lose you must pay 5 dollars. Given your wisdom from taking 321, will you play this game? That is, is there a strategy the expectation of whose return is positive (so it is worth your while using that strategy to play the game)? Why, or why not? (If you claim there is no such strategy, prove your answer, and if you claim there is such a positive yield strategy, give one!)

11. (**Extra Credit**) There is an event in an n-seat auditorium and n shy people have each been assigned one of the n seats. Unfortunately, the first person to walk in is the Absent-Minded Professor, who chooses a seat at random and sits there. People walk in one by one, and each person who walks in after the professor sits down in his or her own seat if it is open. However, if someone is already sitting there, s/he is too shy to protest, and picks an empty seat at random and sits there. What is the probability the last person will sit in his/her assigned seat? What is the expected number of people that end up in their assigned seat?