CSE 321: Discrete Structures — Assignment #3
October 12, 2006 — Due: Friday, October 19

Reading Assignment: 6th Edition: sections 2.1–2.3 & 3.5–3.6 (or, 5th Edition: sections 1.6–1.8 & 2.4–2.5).

Note: If you feel comfortable, you may prove things (when required) in prose form, but your arguments must be rigorous!

Problems:

1. Prove that if you pick 10 numbers from 1 to 1000, then there is a pair of numbers such that the larger of the two is at most twice the other.

2. Prove that
$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1.$$
Hint: Define $S = \sum_{i=0}^{k} 2^i$ and consider the quantity $2S$.

3. Let $Q(A, B)$ be the proposition that $A \subseteq B$. If the universe of discourse for both A and B is all sets of integers, what are the truth values of the following? Justify your answers.
   (a) $\forall A \, \exists B \, Q(A, B)$
   (b) $\forall B \, \exists A \, Q(A, B)$
   (c) $\exists A \, \forall B \, Q(A, B)$
   (d) $\exists B \, \forall A \, Q(A, B)$

4. Carefully prove the following implications
   (a) $(A \cup B = B) \longrightarrow (A \subseteq B)$
   (b) $(A \subseteq B) \longleftrightarrow (\bar{B} \subseteq \bar{A})$

5. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ which is
   (a) one-to-one but not onto
   (b) onto but not one-to-one
   (c) both onto and one-to-one, but not the identity function
   (d) neither one-to-one nor onto

6. How many zeros are there at the end of 100! (Recall that "n factorial" is the number $n! = n(n-1)(n-2)\cdots 2 \cdot 1$)
   Hint: Think about the unique factorization of 100! into primes.

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7. For the purpose of this problem, a word is a finite sequence of lower case letters, like "whatsthefrequencykenneth" or "zzvvyarrrrghomgomg." Let $\mathcal{W}$ be the set of all words. Does $\mathcal{W}$ have the same cardinality as $\mathbb{N}$? (i.e. is there a bijection from one set to the other?) Explain your answer.

8. **Extra credit:** Prove that $\sqrt{n}$ is irrational if and only if n is not a perfect square.

9. **Extra credit:** If functions f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer. (You may need to look up the definition of function composition in the book.)