CSE 321: Discrete Structures | Assignment #6
November 9th, 2007 | Due: **Monday**, November 19th

**Reading Assignment:** 6th Edition: 5.1–5.5 (or, 5th Edition: 4.1–4.5).

**Problems:**

1. In the town of Yreka, California there are three elected positions: mayor, town baker, and master palindromist, to be chosen from among the 78 people in town. Each person can be elected to at most one of these positions
   (a) How many ways are there of filling all three of these positions? That is, how many different election outcomes are possible (assuming no ties)?
   (b) During the election four years ago there were only 67 people in town and someone forgot to prohibit the same person from being elected to more than one position. How many outcomes were possible then?
   (c) After a recent scandal involving a shotgun, a dog, and a gun carrier outfitted for a dog, it was decided that, for the next election, H.E. Shothimself, one of the townspeople in Yreka, will no longer eligible to be chosen as mayor. In how many ways can the three positions be chosen next time?

2. Solve the following counting problems. In each case show the reasoning that led you to the answer.
   (a) A palindrome is a word that reads the same forwards and backwards. How many seven-letter palindromes can be made from the English alphabet?
   (b) How many bit strings of length 10 begin with three 0s or end with two 1s?
   (c) How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?
   (d) How many truth tables are there for propositions with $n$ variables?
   (e) Suppose you have $n$ beads, each of a different color, that you need to string into a necklace? How many distinct necklaces can you make? (A necklace flipped over remains the same and does not count as a distinct necklace.)

3. Yreka, California, being a town founded by mathematicians, is a town with East-West streets numbered 1 through $n$ and North-South Avenues numbered 1 through $m$. A taxi cab picks you up at the corner of 1st Avenue and 1st street, and you wish to be dropped off at the corner of $m$th avenue and $n$th street. Since you are a smart 321 student, you are obviously not going to permit the cab driver to drive longer than the necessary $(n-1)+(m-1)$ blocks and charge you more than the minimum fare. In other words, the cabby must always be increasing his street number or his avenue number. Suppose there is an accident at the intersection of street $i$ and avenue $j$, for some $i$,$j$ where $1 < i < n$ and $1 < j < m$. How many routes can the cab driver take to get you to your destination while avoiding the intersection with the accident? Justify your answer.

4. In a dinner party with $n$ people, all of them are seated at a circular table. Suppose there is a nametag at each place at the table and suppose that nobody sits down in their correct place. Show that it is possible to rotate the table so that at least two people are sitting in the correct place.

5. Prove that

$$\sum_{i=1}^{n} i \binom{n}{i} = n2^{n-1}$$

using a combinatorial proof (i.e. a proof that uses counting arguments to prove that both sides of the identity count the same objects but in different ways.) Hint: Count in two ways the number of ways to select a committee and then select a leader of the committee.

6. **EXTRA CREDIT** Assume that friendship is always mutual; that is, if A is a friend of B then B is also a friend of A. Show that under this assumption in any group of people there are always two people who have exactly the same number of friends within the group.