CSE 321: Discrete Structures | Assignment #8
December 1st, 2007 | Due: **Friday**, December 7th

Reading Assignment: 6th Edition: 8.1,8.3-8.5,9.1-9.5,9.7 (or, 5th Edition: 7.1,7.3-7.5,8.1-8.5,8.7).

Problems:

1. For the relation $R = \{(b,c),(b,e),(c,e),(d,a),(e,b),(e,c)\}$ on $\{a,b,c,d,e\}$, draw the following relations in directed graph form:
   (a) The reflexive closure of R.
   (b) The symmetric closure of R.
   (c) The transitive closure of R.
   (d) The reflexive, symmetric, transitive closure of R.

2. Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation. (Note that this is a relation on a set of ordered pairs. Don't get confused and think that the ordered pairs by themselves are a relation.)

3. Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with is even. Assume that no one shakes his or her own hand.

4. Prove that any (simple, undirected) graph on $n \geq 2$ vertices contains two vertices of equal degree.

5. **Extra credit:** For undirected simple graphs, prove that if G is disconnected, then $\bar{G}$, the complement of G, is connected. (Recall that $\bar{G}$ contains all and only those edges that are absent in G.)

6. **Extra credit:** Suppose that G is a simple, undirected graph and every vertex of G has degree at least d for some $d > 2$. Prove that G must contain a (simple) cycle of length at least $d + 1$.