

Homework 1, Due Wednesday, January 16, 2008

**Problem 1:**

Section 1.1, Exercise 10.

**Problem 2:**

Section 1.1, Exercise 16. (Fifth edition, 1.1, Exercise 14)

**Problem 3:**

Section 1.1, Exercise 20, a, c, e, g. (Fifth edition, 1.1, Exercise 18, a, c, e, g)

**Problem 4:**

Section 1.1, Exercise 34 (Fifth edition, 1.1, Exercise 30).

**Problem 5:**

The following two statements form the basis of the most important methods of theorem proving. Use truth tables to prove that they are tautologies.

- a) Resolution:  $((p \vee q) \wedge (\neg q \vee r)) \rightarrow (p \vee r)$
- b) Modus ponens:  $((p \wedge (p \rightarrow q)) \rightarrow q)$

**Problem 6:**

Show that Modus ponens is a tautology without using a truth table. Show each step and indicate which logical equivalences you use.

**Problem 7:**

Show that  $(p \rightarrow r) \wedge (q \rightarrow r)$  and  $(p \vee q) \rightarrow r$  are logically equivalent.

**Problem 8:**

Define the NAND operator, denoted  $|$ , as follows  $(p | q)$  is true when either  $p$  or  $q$ , or both are false; and  $(p | q)$  is false when  $p$  and  $q$  are true.

- a) Show that the NAND operator is commutative.
- b) Show that the NAND operator is not associative.
- c) Give an expression that is equivalent to  $p \rightarrow q$  that only uses the logical connective  $|$ .

**Extra Credit 9:**

Show that you can swap a pair of memory registers using exclusive-or without any temporary storage. To be precise, suppose that your machine has two memory registers  $R_1$  and  $R_2$  with the same number of bits. The instruction  $XOR_1$  makes the assignment  $R_1 \leftarrow R_1 \oplus R_2$  and the instruction  $XOR_2$  makes the assignment  $R_2 \leftarrow R_1 \oplus R_2$ . Describe how you swap a pair of values and explain why your solution works.