

Homework 5, Due Wednesday, February 13, 2008

Problem 1:

Prove that $n! < n^n$ where n is an integer greater than 1.

Problem 2:

Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

Problem 3:

Casting out nines: Prove that a number is divisible by 9 if and only if the sum of its digits is divisible by 9. More specifically, for a positive integer a , let $digitsum(a)$ be the sum of the decimal digits of a . Prove by induction on the number of decimal digits in a that $a \equiv digitsum(a) \pmod{9}$.

Hint: Express a as $\sum_{i=0}^n a_i 10^i$ where $a_i \in \{0, \dots, 9\}$.

Problem 4:

Use strong induction to show that a rectangular $2n \times 2m$ checkerboard with two squares missing, one white and one black, can be covered with dominoes.

Problem 5:

Section 4.3, Problem 12. (Fifth edition, Section 3.4, Problem 12.)

Extra Credit 6:

Section 4.1, Problem 51. (Fifth edition, Section 3.3, Problem 39)