CSE 321 Discrete Structures

Winter 2008
Lecture 5
Rules of Inference

Announcements

- Reading for this week
  - Today: 1.5
  - Friday: 1.6
- Homework 2
  - Due Wednesday, January 23
- January 21, No class

Highlights from Lecture 4

- Translation between Predicate Calculus and English
- "All red cats like tofu"
  - $\forall x$ ((Cat(x) $\wedge$ Red(x)) $\rightarrow$ LikesTofu(x))
  - $\forall x$ (Cat(x) $\rightarrow$ (Red(x) $\rightarrow$ LikesTofu(x)))
- Nested Quantifiers

Quantification with two variables

Valid Arguments

- Classical logic
- Artificial Intelligence: Automated Reasoning

Reasoning

- "If Seattle won last Saturday they would be in the playoffs"
- "Seattle is not in the playoffs"
- Therefore . . .

Proofs

- Start with hypotheses and facts
- Use rules of inference to extend set of facts
- Result is proved when it is included in the set

Rules of Inference

$$\frac{p \quad p \to q}{\therefore q} \qquad \frac{\neg q \quad p \to q}{\therefore \neg p} \qquad \frac{p \to q \quad q \to r}{\therefore p \to r} \qquad \frac{p \vee q \quad \neg p}{\therefore q}$$

$$\frac{p}{\therefore p \vee q} \qquad \frac{p \quad q}{\therefore p \vee q} \qquad \frac{p \wedge q}{\therefore p} \qquad \frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$$

$$\frac{\forall x\, P(x)}{\therefore P(c)} \qquad \frac{P(c) \text{ for any } c}{\therefore \forall x P(x)} \qquad \frac{\exists x\, P(x)}{\therefore P(c) \text{ for some } c} \qquad \frac{P(c) \text{ for some } c}{\therefore \exists x\, P(x)}$$

Example 6

- Hypotheses
  - It is not sunny this afternoon and it is colder than yesterday
  - We will go swimming only if it is sunny
  - If we do not go swimming, then we will take a canoe trip
  - If we take a canoe trip, we will be home by sunset
- Show:
  - We will be home by sunset

Classical Logic

- All men are mortal
- Socrates is a man
- Therefore, Socrates is mortal

Example 13

- Show "A student in this class has not read the book", and "Everyone in this class passed the exam" imply "Someone who passed the exam has not read the book"

C(x): x is in the class
B(x): x has read the book
P(x): x passed the exam