CSE 321 Discrete Structures

Winter 2008
Lecture 6
Proofs

Announcements

- Reading for this week
  - Today: 1.6, 1.7
- Homework 2
  - Due Wednesday, January 23
- Martin Luther King Jr. Day, Mon., Jan 21
  - $\forall x$ (UniversityHoliday(x) $\rightarrow$ NoClass(x))
  - UniversityHoliday(Monday)

Highlights from Lecture 5

- Formal Reasoning
- Build a proof, starting from hypotheses by applying rules of inference

Proofs

- Proof methods
  - Direct proof
  - Contrapositive proof
  - Proof by contradiction
  - Proof by equivalence

Direct Proof

- If n is odd, then n^2 is odd

Definition
n is even if $n = 2k$ for some integer k
n is odd if $n = 2k+1$ for some integer k

Contrapositive

- Sometimes it is easier to prove $\neg q \rightarrow \neg p$ than it is to prove $p \rightarrow q$
- Prove that if $n \leq ab$ then $a \leq n^{1/2}$ or $b \leq n^{1/2}$

Proof by contradiction

- Suppose we want to prove p is true.
- Assume p is false, and derive a contradiction

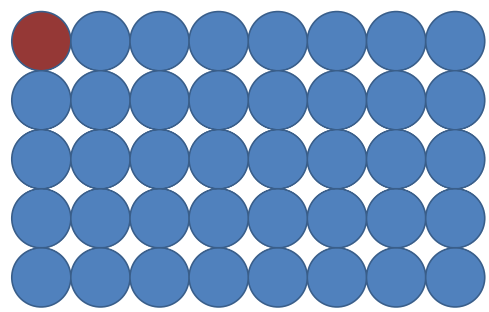
Contradiction example

- Show that at least four of any 22 days must fall on the same day of the week

Equivalence Proof

- To show $p_1 \leftrightarrow p_2 \leftrightarrow p_3$, we show $p_1 \rightarrow p_2$, $p_2 \rightarrow p_3$, and $p_3 \rightarrow p_1$
- Show that the following are equivalent
  - p_1: n is even
  - p_2: n-1 is odd
  - p_3: n^2 is even

The Game of Chomp

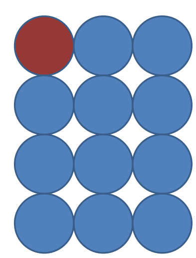


Theorem: The first player can always win in an $n \times m$ game

- Every position is a forced win for player A or player B (this fact will be used without proof)
- Any finite length, deterministic game with no ties is a win for player A or player B under optimal play

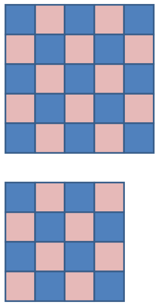
Proof

- Consider taking the lower right cell
  - If this is a forced win for A, then done
  - Otherwise, B has a move m that is a forced win for B, so if A started with this move, A would have a forced win



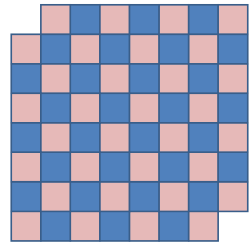
Tiling problems

- Can an $n \times n$ checkerboard be tiled with 2×1 tiles?



8×8 Checkerboard with two corners removed

- Can an 8×8 checkerboard with upper left and lower right corners removed be tiled with 2×1 tiles?



8×8 Checkerboard with one corner removed

- Can an 8×8 checkerboard with one corner removed be tiled with 3×1 tiles?