# CSE 321 Discrete Structures

Winter 2008
Lecture 7
Set Theory and Operations

## Announcements

- Reading for this week
  - Today: 2.1, 2.2 (5th Edition: 1.6, 1.7)
  - Thursday: 2.3 (5th Edition: 1.8 )
  - Friday: 3.4, 3.5 (5th Edition: 2.4, 2.5)
- Homework 3
  - Due Wednesday, January 30
  - Note: problems are not necessarily of the same degree of difficulty

## Highlights from Lecture 6

- Direct Proofs
- Chomp!
- Challenges
  - Develop optimal strategy for 6 $\times$ 8 Chomp!
  - Create a Chomp! program that uses an optimal algorithm
  - Generalizations of Chomp!

## Set Theory

- Formal treatment dates from late 19th century
- Direct ties between set theory and logic
- Important foundational language

## Definition: A set is an unordered collection of objects

## Definitions

- A and B are *equal* if they have the same elements

$$A = B \equiv \forall x\, (x \in A \leftrightarrow x \in B)$$

- A is a *subset* of B if every element of A is also in B

$$A \subseteq B \equiv \forall x\, (x \in A \rightarrow x \in B)$$

## Empty Set and Power Set

## Cartesian Product : $A \times B$

$A \times B = \{ (a, b) \mid a \in A \wedge b \in B\}$

## Set operations

$A \cup B = \{ x \mid x \in A \vee x \in B \}$

$A \cap B = \{ x \mid x \in A \wedge x \in B \}$

$A - B = \{ x \mid x \in A \wedge x \notin B \}$

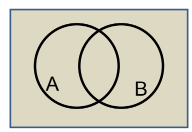
$A \oplus B = \{ x \mid x \in A \oplus x \in B \}$

$\overline{A} = \{ x \mid x \notin A \}$

## De Morgan's Laws

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

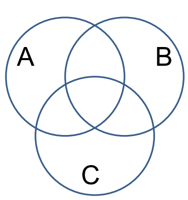
$\overline{A \cap B} = \overline{A} \cup \overline{B}$

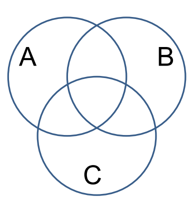



Proof technique:
To show C = D show
$x \in C \rightarrow x \in D$ and
$x \in D \rightarrow x \in C$

## Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$




## Russell's Paradox

$S = \{ x \mid x \notin x \}$