CSE 321 Discrete Structures

Winter 2008
Lecture 19
Probability Theory

Announcements

- Readings
  - Probability Theory
    - 6.1, 6.2 (5.1, 5.2) Probability Theory
    - 6.3 (New material!) Bayes' Theorem
    - 6.4 (5.3) Expectation
  - Advanced Counting Techniques – Ch 7.
    - Not covered

Highlights from Lecture 18

- Experiment
- Sample Space
- Event
- Probability

Combinations of Events

E^C is the complement of E

$P(E^C) = 1 - P(E)$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Probability Concepts

- Probability Distribution
- Conditional Probability
- Independence
- Bernoulli Trials / Binomial Distribution
- Random Variable

Discrete Probability Theory

- Set S
- Probability distribution $p : S \rightarrow [0,1]$
  - For $s \in S$, $0 \leq p(s) \leq 1$
  - $\Sigma_{s \in S}\, p(s) = 1$
- Event E, $E \subseteq S$
- $p(E) = \Sigma_{s \in E}\, p(s)$

Examples

Conditional Probability

Let E and F be events with p(F) > 0. The conditional probability of E given F, defined by p(E | F), is defined as:

$$p(E \mid F) = \frac{p(E \cap F)}{p(F)}$$

Examples

Independence

The events E and F are independent if and only if $p(E \cap F) = p(E)p(F)$

E and F are independent if and only if $p(E \mid F) = p(F)$

Are these independent?

- Flip a coin three times
  - E: the first coin is a head
  - F: the second coin is a head
- Roll two dice
  - E: the sum of the two dice is 5
  - F: the first die is a 1
- Roll two dice
  - E: the sum of the two dice is 7
  - F: the first die is a 1
- Deal two five card poker hands
  - E: hand one has four of a kind
  - F: hand two has four of a kind

Bernoulli Trials and Binomial Distribution

- Bernoulli Trial
  - Success probability p, failure probability q

The probability of exactly k successes in n independent Bernoulli trials is

$$\binom{n}{k} p^k n^{n-k}$$

Random Variables

A random variable is a function from a sample space to the real numbers