CSE 321 Discrete Structures

Winter 2008
Lecture 22
Binary Relations

Announcements

- Readings
  - Relations
    - Section 8.1 Binary Relations
    - Section 8.2 n-Ary relations
    - Section 8.3 Representing Relations

Highlights from Lecture 21

- Expectation of a Random Variable
  - Linearity of Expectation
- Average Case Analysis of Algorithms

Relations

Definition of Relations

Let A and B be sets,
A binary relation from A to B is a subset of $A \times B$

Let A be a set,
A binary relation on A is a subset of $A \times A$

Relation Examples

Properties of Relations

Let R be a relation on A

R is reflexive iff $(a,a) \in R$ for every $a \in A$

R is symmetric iff $(a,b) \in R$ implies $(b, a) \in R$

R is antisymmetric iff $(a,b) \in R$ and $a \neq b$ implies $(b,a) \notin R$

R is transitive iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Combining Relations

Let R be a relation from A to B
Let S be a relation from B to C
The composite of R and S, $S \circ R$ is the relation from A to C defined

$S \circ R = \{(a, c) \mid \exists\ b$ such that $(a,b) \in R$ and $(b,c) \in S\}$

Examples

$(a,b) \in$ Parent: b is a parent of a
$(a,b) \in$ Sister: b is a sister of a
What is Parent $\circ$ Sister?

What is Sister $\circ$ Parent?

$S \circ R = \{(a, c) \mid \exists\ b$ such that $(a,b) \in R$ and $(b,c) \in S\}$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother express

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Powers of a Relation

$R^2 = R \circ R = \{(a, c) \mid \exists\ b$ such that $(a,b) \in R$ and $(b,c) \in R\}$

$R^0 = \{(a,a) \mid a \in A\}$

$R^1 = R$
$R^{n+1} = R^n \circ R$

How is  related to  ?

From the Mathematics Geneology Project

Erhard Weigel
Gottfried Leibniz
Jacob Bernoulli
Johann Bernoulli
Leonhard Euler
Joseph Lagrange
Jean-Baptiste Fourier
Gustav Dirichlet
Rudolf Lipschitz
Felix Klein
C. L. Ferdinand Lindemann
Herman Minkowski
Canstantin Caratheodory
Georg Aumann
Friedrich Bauer
Manfred Paul
Ernst Mayr
Richard Anderson

Transitivity and Composition

R is Transitive if and only if $R^n \subseteq R$ for all $n \geq 1$

n-ary relations

Let $A_1, A_2, \ldots, A_n$ be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \ldots \times A_n$.

Relational databases

Student_Name	ID_Number	Major	GPA
Knuth	328012098	CS	4.00
Von Neuman	481080220	CS	3.78
Von Neuman	481080220	Mathematics	3.78
Russell	238082388	Philosophy	3.85
Einstein	238001920	Physics	2.11
Newton	1727017	Mathematics	3.61
Karp	348882811	CS	3.98
Newton	1727017	Physics	3.61
Bernoulli	2921938	Mathematics	3.21
Bernoulli	2921939	Mathematics	3.54

Alternate Approach

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328012098	CS
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348882811	CS
1727017	Physics
2921938	Mathematics
2921939	Mathematics

Database Operations

Projection

Join

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