CSE 321 Discrete Structures

Winter 2008
Lecture 23
Relations

Announcements

- Readings
  - Today
    - Section 8.2 n-Ary relations
    - Section 8.3 Representing Relations
  - Friday (Natalie)
    - 8.4 Closures (Key idea – transitive closure)
    - 8.5 Equivalence Relations (Skim)
    - 8.6 Partial Orders
  - Next week
    - Graph theory

Highlights from Lecture 22

Let A and B be sets,
A binary relation from A to B is a subset of $A \times B$

Composition

$S \circ R = \{(a, c) \mid \exists\ b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$

Transitivity

$(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Transitivity and Composition

R is transitive if and only if $R^n \subseteq R$ for all $n \geq 1$

n-ary relations

Let $A_1, A_2, \ldots, A_n$ be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \ldots \times A_n$.

Relational databases

Student_Name	ID_Number	Major	GPA
Knuth	328012098	CS	4.00
Von Neuman	481080220	CS	3.78
Von Neuman	481080220	Mathematics	3.78
Russell	238082388	Philosophy	3.85
Einstein	238001920	Physics	2.11
Newton	1727017	Mathematics	3.61
Karp	348882811	CS	3.98
Newton	1727017	Physics	3.61
Bernoulli	2921938	Mathematics	3.21
Bernoulli	2921939	Mathematics	3.54

Alternate Approach

Student_ID	Name	GPA
328012098	Knuth	4.00
481080220	Von Neuman	3.78
238082388	Russell	3.85
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1727017	Newton	3.61
348882811	Karp	3.98
2921938	Bernoulli	3.21
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328012098	CS
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348882811	CS
1727017	Physics
2921938	Mathematics
2921939	Mathematics

Database Operations

Projection

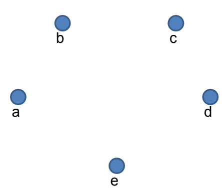
Join

Select

Representation of relations

Directed Graph Representation (Digraph)

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }



Matrix representation

Relation R from A={a_1, … a_p} to B={b_1, . . . b_q}

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

{(1, 1), (1, 2), (1, 4), (2,1), (2,3), (3,2), (3, 3) }

Matrix operations

How do you tell if a relation is reflexive from its adjacency matrix?

How do you tell if a relation is symmetric from its adjacency matrix?

Suppose R has matrix M_R and S has Matrix M_S.
What are the matrices for R∪ S and R∩ S?

Matrix multiplication

Standard (×, +) matrix multiplication.
A is a m × n matrix, B is a n × p matrix
C = A × B is a m × p matrix defined:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

And-OR Matrix multiplication

A is a m × n boolean matrix, B is a n × p boolean matrix
C = A ⊗ B is a m × p matrix defined:

$$c_{ij} = \bigvee_{k=1}^{n} (a_{ik} \wedge b_{kj})$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} (a_{11} \wedge b_{11}) \vee (a_{12} \wedge b_{21}) \vee (a_{13} \wedge b_{31}) & (a_{11} \wedge b_{12}) \vee (a_{12} \wedge b_{22}) \vee (a_{13} \wedge b_{32}) & \cdots \\ (a_{21} \wedge b_{11}) \vee (a_{22} \wedge b_{21}) \vee (a_{23} \wedge b_{31}) & (a_{21} \wedge b_{12}) \vee (a_{22} \wedge b_{22}) \vee (a_{23} \wedge b_{32}) & \cdots \\ (a_{31} \wedge b_{11}) \vee (a_{32} \wedge b_{21}) \vee (a_{33} \wedge b_{31}) & (a_{31} \wedge b_{12}) \vee (a_{32} \wedge b_{22}) \vee (a_{33} \wedge b_{32}) & \cdots \end{bmatrix}$$

Matrices and Composition

$M_{S \circ R} = M_R \otimes M_S$

R = {(a, a), (a, c), (b, a), (b, b)}
S = {(b, a), (b, c), (c, a), (c, c)}