321 Section, Week 3

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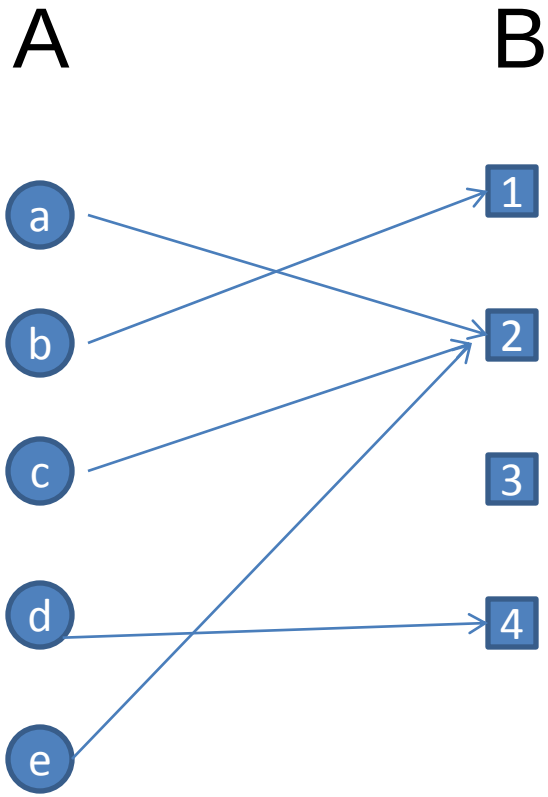
Functions

- A **function** from *A* to *B* is an assignment of exactly one element of *B* to each element of *A.* We write *f(a) = b* if *b* is the unique element of *B* assigned by *f* to the element *a* of *A.* If *f* is a function from *A* to *B*, we write *f: A→B.* We say that *f* maps *A* to *B*
- **Domain**: A
- **Codomain**: B
- If *f(a) = b, b* is the **image** of *a*, *a* is the **preimage** of *b*
- **Range** of *f* is the set of all images of elements of A

Identify the domain, codomain, range, image of a, preimage of 2 for the following



What does it mean for a function to be one-to-one (aka injective)?

- Use a picture

F(a) = f(a')->a=a'

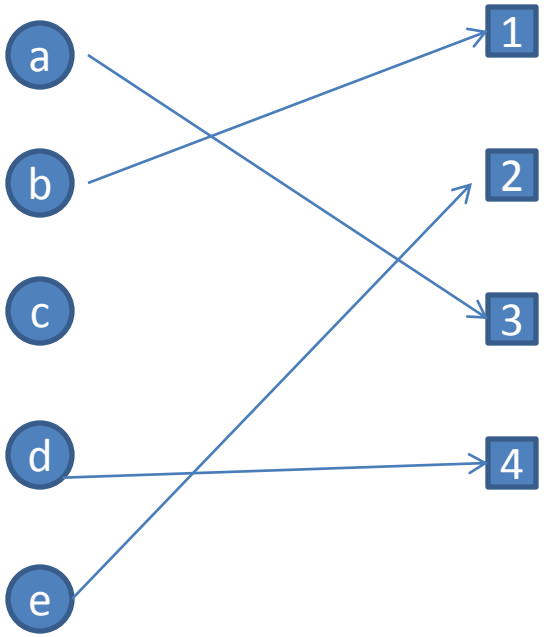
What does it mean for a function to be onto (aka surjective)?

Every b has an a f(a)=b Mention that a bijection is both

Is this -a function?

-one-to-one?

-onto?



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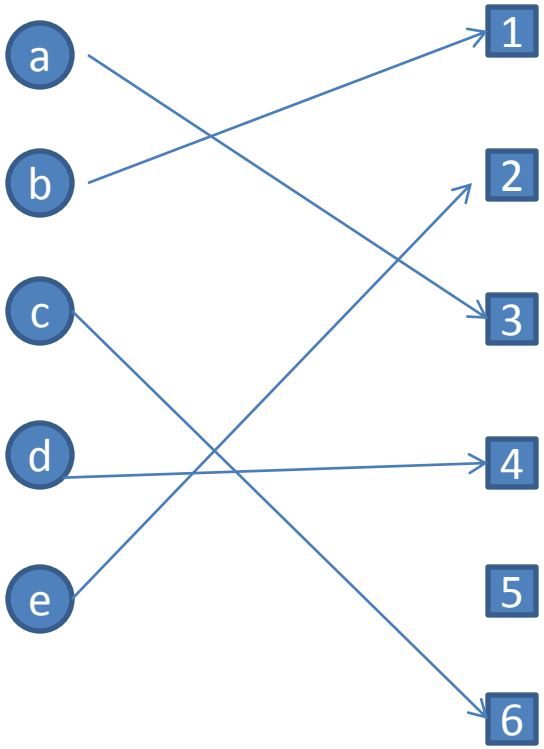

What’s an inverse?

F(a) = b -> f-1(b) = a

What's the inverse of

- x
- x^2

What kinds of functions have inverses?

bijections

What's the composition of two functions?

- Draw a picture

Fog(a) = f(g(a))

A few more things that you probably already know

- Increasing
- Strictly increasing
- Decreasing
- Strictly decreasing
- Product
- Sum
- Ceiling function
- Floor function

Homework 1

- Associativity (parentheses matter!)

HW1

- Proof style
- Only if

Fallacies

- Affirming the conclusion

- Denying the hypothesis

P->q, q, therefore p;;p->q, -p, therefore -q

Machine representation of sets

- Store the set somewhere in a given order
- Represent a subset by a sequence of zeros and ones that express subset membership
- U = {1,2,3,4,5,6,7,8,9,10}
- Subset {1,2,3} = 1110000000

Represent subsets of {1,2,3,4,5,6,7,8,9,10}

- Odd numbers
- Even numbers
- How do you find the complement of a subset given its binary representation?

Union and intersection

- How would you compute the union of 1010101010 and 1111100000?

  How would you compute the intersection of 1010101010 and 1111100000

Use a direct proof to prove that the product of two rational numbers is rational

$$(A \cup B) \subseteq (A \cup B \cup C)$$

$$(B-A) \cup (C-A) = (B \cup C) - A$$

Use a direct proof to show that every odd integer is the difference of two squares

K+1, k; 1.6 #7