$$\left[P(1) \wedge \forall k\ \underline{P(k)} \to P(k+1) \right] \to \forall n\ P(n)$$

$$\overbrace{1 + \underbrace{2 + 3 + \cdots}_{} + 100}$$

$$P(n)\left[\sum_{1}^{n} i = \frac{n(n+1)}{2} \right]$$

Basis $P(1) \equiv 1 = \dfrac{1(1+1)}{2} = 1$ ✓

$$\underbrace{1 + 2 + 3 \cdots + n}_{} + n + 1$$

$$= \frac{n(n+1)}{2} \quad \text{by ind hypothesis}$$

$$= \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

$P(n+1)$

by ind. $\forall n\ P(n)$

$$H_k = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{k} \qquad \text{"}k^{th} \text{ harmonic number"}$$

$$\boxed{H_{2^n} \geq 1 + n/2 \quad \forall n \geq 0} \qquad P(n)$$

Basis

$$H_{2^0} = H_1 = \frac{1}{1} = 1 \geq 1 + \frac{0}{2}$$

$P(0)$ holds

Ind. step

$$H_{2^{k+1}} = \underbrace{\overbrace{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{2^k}}^{2^k}}_{H_{2^k} \geq 1 + k/2} + \underbrace{\overbrace{\frac{1}{2^k+1} + \frac{1}{2^k+2} \cdots \frac{1}{2^{k+1}}}^{2^k}}_{\geq 2^k \cdot \frac{1}{2^{k+1}} = 1/2}$$

$$\geq 1 + \frac{k}{2} + \frac{1}{2} = 1 + \frac{k+1}{2}$$

So $P(k+1)$ holds if $P(k)$ does

$\therefore$ by principle of induction $\forall n \geq 0 \; P(n)$

Every integer $\underline{n \geq 2}$ is a product of primes
$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{P(n)}$

~~Basis~~ $P(2)$

$2 = 2$, 2 is prime

<u>Induction</u>

$n \geq 2$

if n is prime, finished

if n is composite, say $n = a \cdot b$

$n > a, b$ ints ≥ 2

want a is product of primes $p_1 \cdot p_2 \cdot p_3 \cdots p_k$

b . . . . . . — $q_1 \cdots q_l$

then $n = p_1 \cdots p_k q_1 \cdots q_l$

Strong induction

$\left[P(1) \wedge \forall_n \left(\forall k \leq n \; P(k)\right) \rightarrow P(n+1)\right] \rightarrow \forall_n P(n)$

Game

players take any # >0 of stones from <u>one</u> pile

last move wins.

$P(n)$: 2^{nd} wins game starting with 2 piles of n.

Basis: $n=1$

Player 1 picks a stone

" 2 ... remaining & wins

Ind.

assume play 2 has winning strategy on any game starting with $< n$ stones per pile

— if P1 takes all, P2 does same & wins

" " " " " " part, " " " " & by ind. wins