$$n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1$$

\+ simple

\- vague

---

$f(0) = 1$

$f(n) = n \cdot f(n-1)$ for all integers $n > 1$

$f(1) = 1$

$f(2) = 2 \cdot f(1) = 2 \cdot 1$

$f(3) = 3 \cdot f(2) = 3 \cdot 2 \cdot 1$

$\vdots$

\+ precise

\- (little more complex)

# Fibonacci

$f_0 = 0$

$f_1 = 1$

$f_n = f_{n-1} + f_{n-2}$ for all integers $n \geq 2$

$f_2 = f_1 + f_0 = 1 + 0 = 1$

$f_3 = f_2 + f_1 = 1 + 1 = 2$

$f_4 = f_3 + f_2 = 2 + 1 = 3$

$f_5 = f_4 + f_3 = 3 + 2 = 5$

$\vdots$

conjecture

$$r^{n-1} \geq f_n \geq r^{n-2} \qquad \text{for some } r > 1$$

all $n \geq 2$

$$f_n = f_{n-1} + f_{n-2} \geq r^{n-3} + r^{n-4}$$

$$= r^{n-4}(1+r) \stackrel{?}{=} r^{n-2}$$

true if $r^{n-4}(1+r) = r^{n-2}$

$$1 + r = r^2$$

$$r^2 - r - 1 = 0$$

$$r = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\approx \begin{array}{l} -.68 \\ +1.7\ldots \end{array}$$

$$r = \frac{1+\sqrt{5}}{2}$$

# A well-formed formula (WFF)

$p \vee q \wedge \neg r$ $\quad (p \vee (q \wedge (\neg r)))$

$p \wedge \vee q \neg$

Defn

1. $\underline{T}$, $\underline{F}$, $\underline{p}$ for some propositional variable $p$ are WFF

2. if $E$ & $F$ are WFF's then so are
   (a) $\underline{(\neg E)}$
   (b) $\underline{(E \vee F)}$
   (c) $\underline{(E \wedge F)}$


$(p \vee (q \wedge (\neg r)))$

$v$ $\quad$ T $\quad$ F $\quad$ T

given function $v$

st $v(T) = T$

$v(F) = F$

$v(p) =$ one or the other

for all prop. variables $p$

Defn. For a wff $E$, $V(E)$

is:

$v(E)$ if $E$ is $T$, $F$, $p$

$\neg V(G)$ if $E$ is $(\neg G)$

$V(G) \vee V(F)$ if $E = (G \vee F)$

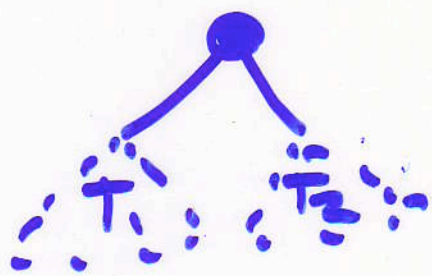
$V(G) \wedge V(F)$ if $E = (G \wedge F)$

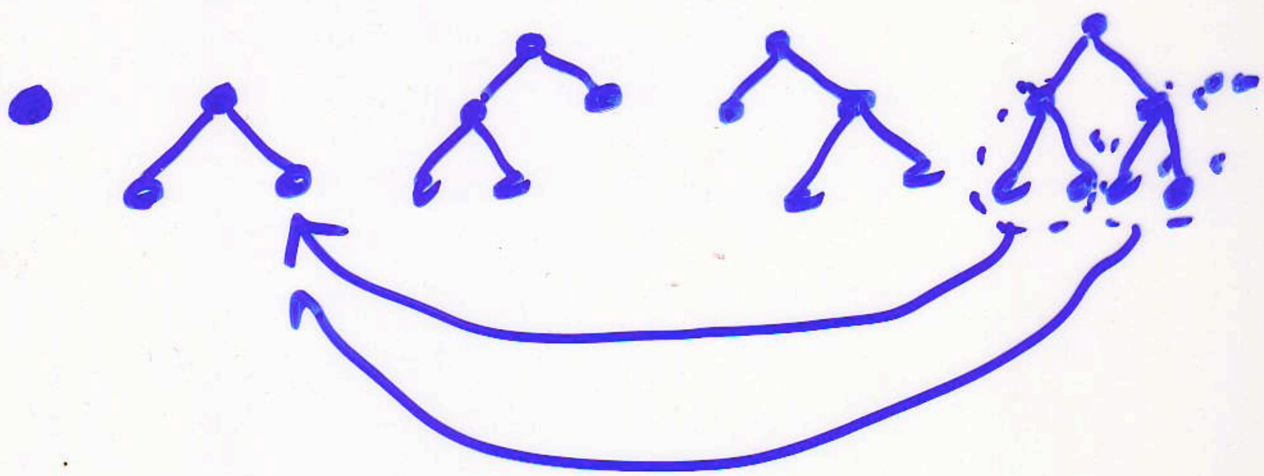
# Full binary Trees

is either

1) a node by itself •

or 2) a node joined to two other full binary trees



$n(T) = 1$ if $T$ is •

$\quad = 1 + n(T_1) + n(T_2)$

$h(T) = 0$ if $T$ is •

$\quad = 1 + \max(h(T_1), h(T_2))$

$$n(T) \le 2^{h(T)+1} - 1$$

if $T = \bullet$ $\quad n(T) = 1$

$h(T) = 0$

$$2^{0+1} - 1 = 1 \quad \checkmark$$

$T =$ 


$$n(T) = 1 + n(T_1) + n(T_2)$$

$$\le 1 + 2^{h(T_1)+1} - 1 + 2^{h(T_2)+1} - 1$$

$$\le 2 \cdot 2^{\max(h(T_1), h(T_2))+1} - 1$$

$$2^{\max(h(T_1), h(T_2))+2} - 1$$

$$= 2^{h(T)+1} - 1$$